

# Balance sheet policies in the euro area

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\* The views expressed here do not necessarily reflect those of the European Central Bank.

# Recent central bank policies

- ▶ Major central banks significantly responded to the financial crisis since 2008/2009
- ▶ Conventional policies
  - ▶ Short-term interest rates were reduced to levels close to the zero lower bound
- ▶ Unconventional policies
  - ▶ Fed & BoE: New lending facilities and direct asset
  - ▶ ECB: full allotment LTRO and bond purchases (SMP and announced OMT)
  - ▶ Summarized by *balance sheet policies* (Curdia and Woodford, 2011)
    - Aimed to reduce spreads and ensure functioning of interbank & credit markets

# Research on balance sheet policies

- ▶ Balance sheet policies introduced with hardly any theoretical or modelling guidance
  - ▶ Conventional models are unable to explain their effectiveness
- ▶ Recent studies on unconventional policies
  - ▶ Non-monetary models: Del Negro et al. (2011), Gertler and Karadi (2011), Gertler and Kiyotaki (2011), Gertler et al. (2012), Chen et al. (2012)
  - ▶ Monetary model: Curdia and Woodford (2011) show that bond purchases are ineffective
- ▶ Interactions of monetary and fiscal policy are typically neglected

Introduce a notion of balance sheets into a DSGE model

- ▶ "Modelling monetary policy in the great moderation":  
Central Bank sets  $R \Rightarrow$  marginal rate of intertemporal substitution (MRIS)
- ▶ "recent crisis: 'hampered' transmission mechanism from interest rate decisions on real activity":  
Central Bank sets  $R \Rightarrow$  financial intermediation ?  $\rightsquigarrow$ ? MRIS
- ▶ introduce financial intermediation and central bank liquidity operations to model 'balance sheet policies'

- ▶ Medium scale macroeconomic model (building on Schabert, 2012)
  - ▶ can reasonably be estimated with Bayesian methods (Smets and Wouters, 2007)
- ▶ Households
  - ▶ consume, supply labor, exchange state contingent contracts, and
  - ▶ deposit funds at the financial intermediaries.
- ▶ Firms
  - ▶ borrow from banks to finance up-front payment of wages, and
  - ▶ set prices in an imperfectly flexible way

## ▶ Banks

- ▶ receive deposits, supply loans to firms, and
- ▶ hold government bonds and reserves for liquidity management.

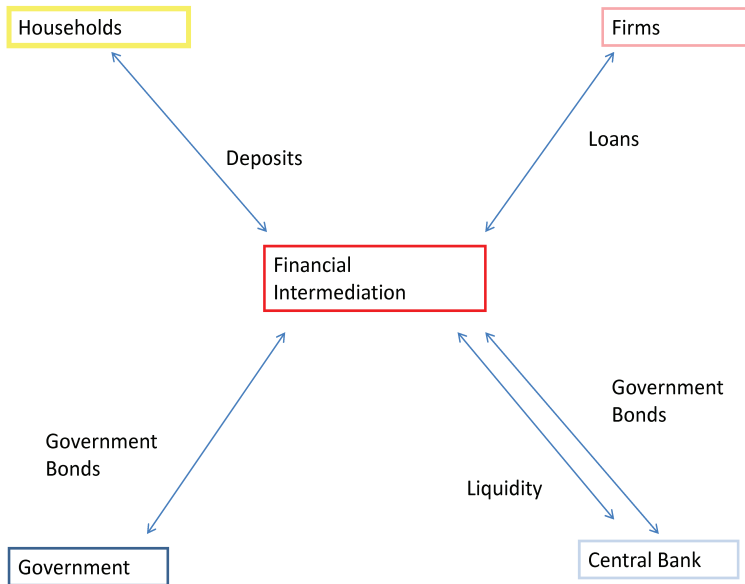
## ▶ Government

- ▶ purchases goods, raises lump-sum taxes, and
- ▶ issues nominal long-term debt as perpetuities

## ▶ Central bank

- ▶ sets the main refinancing rate
- ▶ decides on haircuts and the amount of bond purchases

# Model in nutshell



- ▶ Perfectly competitive *banks*, i.e. financial intermediaries
  - ▶ receive deposits from household  $D_t = \int D_{i,t} di$ , invest in loans  $L_t = \int L_{j,t} dj$
  - ▶ hold government bonds issued at the price  $q_t^B$  in period  $t$  and deliver the payoff  $p_{t+1}^B$  in period  $t + 1$
  - ▶ and demand reserves to manage deposits and loans.
  - ▶ are subject to a balance sheet constraint

$$D_t = M_t + E_t p_{t+1}^B B_t + L_t.$$

- ▶ Only bonds are eligible and are discounted at the main refinancing rate  $R_t^m$  :

$$I_t \leq \kappa_t \cdot p_t^B B_{t-1} / R_t^m, \quad (1)$$

where  $\kappa_t$  allows the central bank to control the terms of lending, e.g. *haircuts*.



- ▶ Financial frictions are specified in a stylized way (Curdia and Woodford, 2011)
  - ▶ Banks face real convex costs when they supply loans
  - ▶ Costs are reduced by holdings of reserves

$$\Xi_t = \Xi \left( \frac{L_t}{P_t}, \frac{M_{t-1} + I_t - \mu D_{t-1}}{P_t}, \zeta_t \right) \geq 0, \quad (2)$$
$$\Xi_{l,t} \geq 0, \Xi_{m,t} \leq 0$$

where  $\zeta_t$  denotes a shock to the banking costs.

- ▶ functional form  $\Xi_t(l_t, i_t) = \zeta_t \left( \frac{l_t}{(m_{t-1}\pi_t^{-1} - \mu d_{t-1}\pi_t^{-1} + i)^\omega} \right)^{\eta_{rc}}$ ,

- ▶ Costs of money equal to  $I_t (R_t^m - 1)$ , such that real profits of a bank  $v_t^I$  satisfy

$$\begin{aligned} \frac{D_t}{R_t^d} - D_{t-1} &\geq q_t^B B_t - p_t^B B_{t-1} + \frac{L_t}{R_t^L} - L_{t-1} & (3) \\ &+ M_t - M_{t-1} + I_t (R_t^m - 1) + P_t \Xi_t + P_t v_t^I, \end{aligned}$$

- ▶ Maximizing present value of profits  $v_t^I$  s.t. (1) and (3) gives

$$\frac{1}{R_{t+k}^d} = 1 + (1 - \mu)E_{t+k}\varphi_{t,t+k+1}\Xi_{m,t+k+1},$$

$$\frac{1}{E_{t+k}R_{t+k+1}^b} = \frac{1}{R_{t+k}^d} + \mu E_{t+k}\varphi_{t,t+k+1}\Xi_{m,t+k+1} + \frac{E_{t+k}R_{t+k+1}^b\varphi_{t,t+k+1}\eta_{t+k+1}\kappa_{t+k+1}}{E_{t+k}R_{t+k+1}^b},$$

$$\frac{1}{R_{t+k}^L} = \frac{1}{R_{t+k}^d} + \mu E_{t+k}\varphi_{t,t+k+1}\Xi_{m,t+k+1} - \Xi_{l,t+k},$$

$$R_{t+k}^m\eta_{t+k} = 1 - R_{t+k}^m - \Xi_{m,t+k},$$

$\eta_t$  is the multiplier on the collateral constraint.

- ▶ Central bank supplies money in open market operations

$$I_t = M_t - M_{t-1} + M_t^R$$

- ▶ Outright  $M_t = \int_0^1 M_{i,t} di$  or via repurchase agreements  
 $M_t^R = \int_0^1 M_{i,t}^R di$
- ▶ Seigniorage  $P_t \tau_t^m$  as interest earnings from repos or asset holdings:

$$P_t \tau_t^m = E_t p_{t+1}^B B_t^c - q_t B_t^c + (R_t^m - 1) (M_t^R + M_t - M_{t-1}) .$$

- ▶ Central bank bond holdings evolve according to

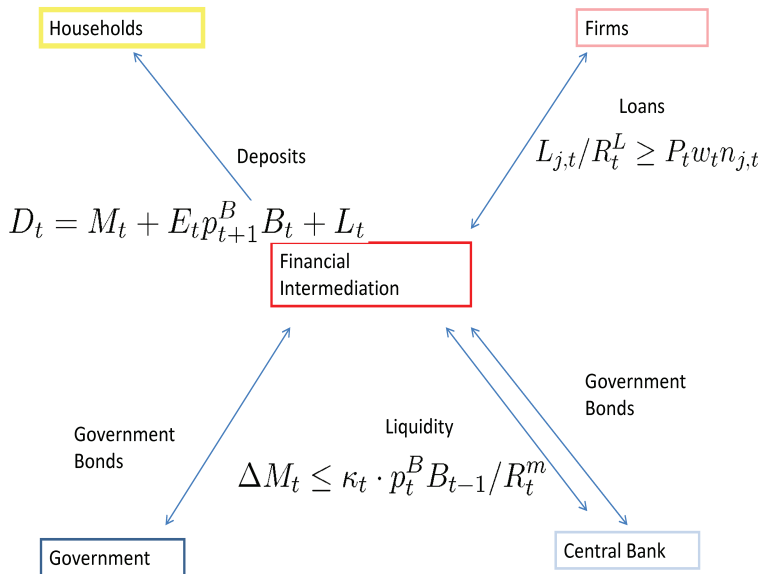
$$q_t B_t^c - p_t^B B_{t-1}^c + P_t \tau_t^m = (M_t - M_{t-1}) R_t^m + M_t^R (R_t^m - 1) .$$

- ▶ Policy rate ("main refinancing rate") is set conventionally (i.i.d. shocks  $\varepsilon_{r,t}$ )

$$R_t^m = (R_{t-1}^m)^{\rho_R} (R^m)^{1-\rho_R} (\pi_t/\pi)^{\rho_\pi(1-\rho_R)} (y_t/y)^{\rho_y(1-\rho_R)} \exp \varepsilon_{r,t}$$

- ▶ CB also chooses how many eligible assets are purchased in period  $t$ , i.e. it sets  $\kappa_t$ 
  - ▶ Full allotment  $\kappa_t = 1$  and *neutralized* money supply  
 $\kappa_t = i \cdot R^m / (p_t^b b_{t-1} \pi_t^{-1})$
  - ▶ Exact paths for  $\kappa_t$  could be identified with data from open market operations
- ▶ Central bank further exogenously sets the fraction of repos  $\Lambda_t > 0 : M_t = \Lambda_t M_t^R$ .

# Model in nutshell



# Central bank independence

- ▶ Steady state consistent with unconditional means for lending and bond rates

$$R^L > R^{Euler} \quad \text{and} \quad \eta_t > 0 \Leftrightarrow R_t^m - 1 < -\Xi_{m,t}.$$

implying binding firms' liquidity constraint and binding collateral constraint of banks.

- ▶ Instrument  $\kappa_t$  allows the central bank to independently control the inflation rate, i.e. adjust (for a given value of eligible assets) access to reserves via settings of open market operations

**Proposition 1** *For a given inflation target and steady state level of public debt, the central bank cannot affect the allocation in a long-run equilibrium via balance sheet policies.*

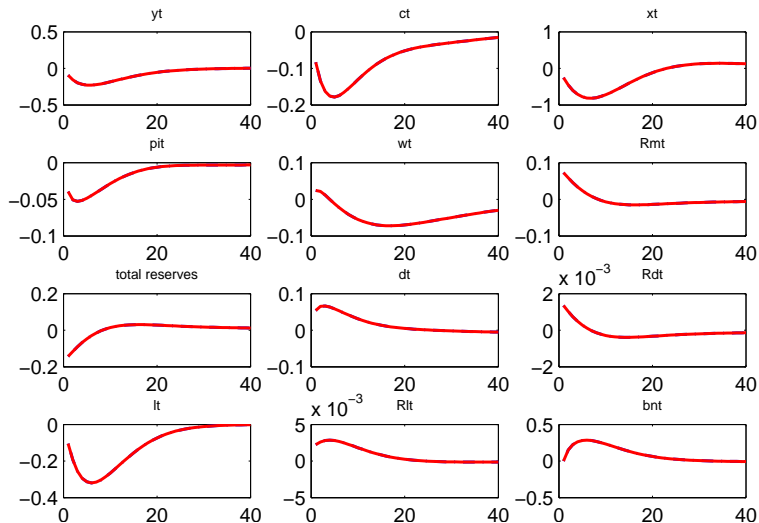
- ▶ but: changes in the size of public debt are in general not neutral

# Calibration/Estimation

- ▶ Model is partly calibrated and estimated with Bayesian techniques.
- ▶ Estimations with quarterly data for the euro area (1981 to 2007)
  - ▶ Nine time series: real GDP, real investment, real private consumption, PC deflator, wage deflator, loans to private sector and respective lending rates, policy rate (main refinancing operations, MRO), total reserves.
  - ▶ Nine shocks: time preference shock ( $\xi_t$ ), a total factor productivity shock ( $a_t$ ), a government spending shock ( $\varepsilon_{gt}$ ), a price mark-up shock ( $\varepsilon_{\pi,t}$ ) a wage mark-up shock ( $\varepsilon_{w,t}$ ) and an interest rate shock ( $\varepsilon_{r,t}$ ), banking cost shock ( $\eta_{rc,t}$ ) and a shock to the collateral constraint ( $\eta_{m,t}$ )
  - ▶ All shocks except the shock to the Taylor rule are modelled as AR(1) shocks.



# Monetary policy shock

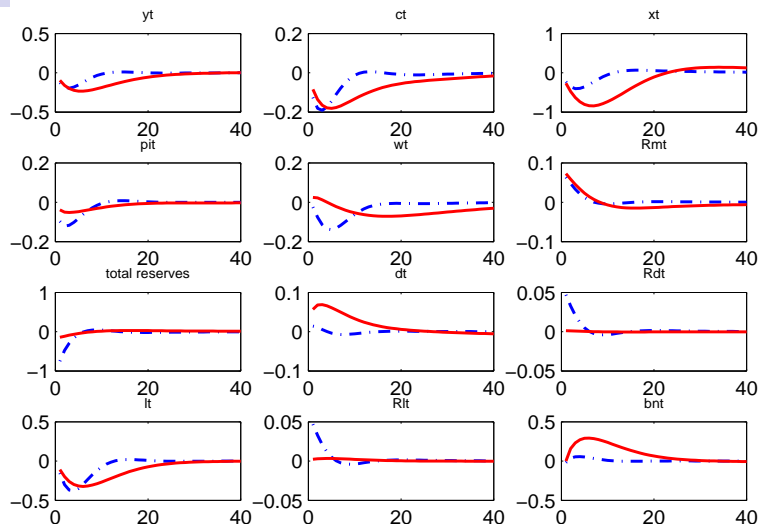


Impulse responses to a monetary policy shock under binding collateral constraint (red solid line)

# Transmission mechanism to monetary policy shock

- ▶ an increase in the policy rate increases the discounting of the collateral
- ▶ financial intermediaries react to lower liquidity by increasing the bond holding and increasing the lending rate and reducing loans
- ▶ less loans at higher lending rate reduce the wage bill firms can pay in advance, lowering wages and employment
- ▶ lower wages and employment lower inflation and aggregate demand

# Monetary policy shock: binding vs non-binding collateral constraint



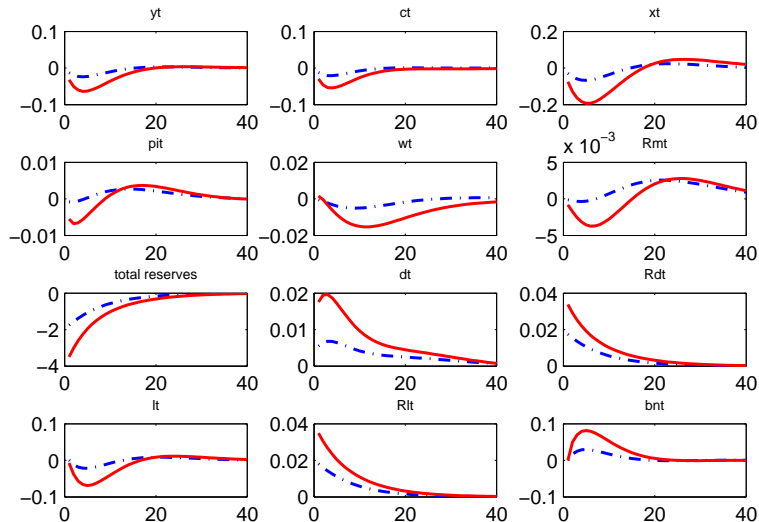
Impulse responses to a monetary policy shock under binding collateral constraint (red solid line), the broken blue line gives the responses under a slack collateral constraint

# Monetary policy shock: binding vs non-binding collateral constraint

The solid line displays the responses to a monetary policy shock under an alternative model with a non-binding collateral constraint

- ▶ under non-binding collateral constraints, the higher discounting of collateral implies higher costs of credit provision which is transmitted to higher lending rates
- ▶ in comparison to the case of a binding collateral constraints the response of the lending rate is more pronounced while the response of bond holdings is less pronounced.
- ▶ under a binding constraint the higher discounting has a direct impact on liquidity which financial intermediaries aim to off-set by buying additional bonds, under a non-binding constraint the optimal response to higher discounting is to reduce reduce liquidity and increase the cost of providing credit.

# Collateral shock: neutralizing balance sheet policy



Impulse responses to a banking cost shock under binding collateral constraint (red solid line), the blue broken line gives the responses under the assumption of a neutralizing balance sheet policy .

## Collateral shock: neutralizing balance sheet policy

The blue broken line gives the responses under the assumption of a neutralizing balance sheet policy

- ▶ a negative collateral shock reduces the value of the collateral of banks  $\Rightarrow$  lower lending at higher cost  $\Rightarrow$  reduction in output
- ▶ the central bank can offset these effects by adjusting the haircut  $\kappa_t$
- ▶ Full allotment  $\kappa_t = 1$  and *neutralized* money supply  
$$\kappa_t = i \cdot R^m / (p_t^b b_{t-1} \pi_t^{-1})$$

## Conclusion: Main results

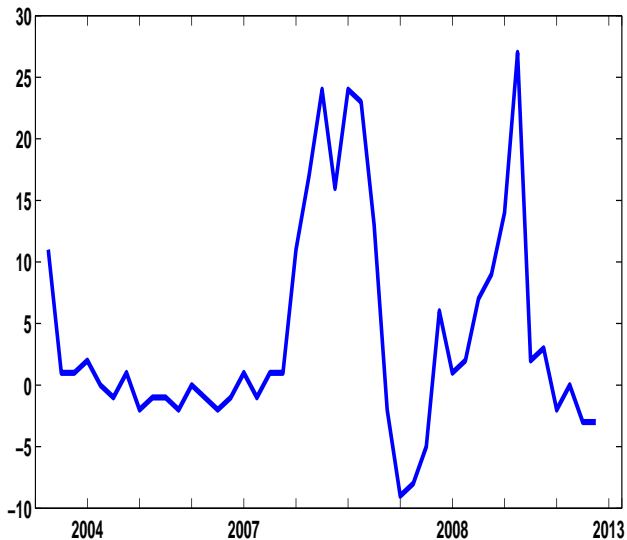
- ▶ the model provides a tool to analyze the effects of balance sheet policies on the macroeconomy.
- ▶ the model has reasonable empirical properties and provides results inline with conventional wisdom on shock transmission.
- ▶ Central bank balance sheet policies can be effective by offsetting negative shocks
- ▶ Changes in the value of available collateral can substantially affect bank lending

# Extensions to this paper

- ▶ distinction between
  - ▶ full commitment: government commits on full repayment debt
  - ▶ limited commitment: lack of commitment on debt repayment (partial default) → value of collateral changes with news on future surplus



# BLS question on liquidity position on credit standards



- ▶ Infinitely lived and identical households indexed with  $i \in [0, 1]$
- ▶ Utility increases with consumption  $c_{i,t}$  and decreases with working time  $n_{i,t}$ 
  - ▶ Deposits  $D_{i,t-1}$  also provide utility (short-cut for modelling transaction services)

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u(\nu_t, c_{i,t}, c_t, n_{i,t}, D_{i,t-1}/P_t),$$

where  $P_t$  is the price of the wholesale good.

- ▶ Time preference shocks  $\xi_t$  and external habits ( $h \cdot c_{t-1}$ ) facilitate model estimation

- ▶ Household  $i$  invest in deposits, and state contingent claims  $S_{i,t}$ ,

$$\left( D_{i,t}/R_t^d \right) - D_{i,t-1} + E_t[\varphi_{t,t+1}S_{i,t+1}] - S_{i,t} + P_t c_{i,t} \leq W_{i,t}n_{i,t} + P_t pr_{i,t} + P_t \tau_{i,t} + P_t \tau_{i,t}^m,$$

where  $R_t^d$  denotes the deposit rate and  $\tau_{i,t}$  a lump-sum tax, and  $pr_{i,t}$  profits.

- ▶ The nominal rate of intertemporal substitution  $R_t^{Euler}$  equals

$$R_t^{Euler} = 1/\varphi_{t,t+1}$$

where and is in general not identical to the monetary policy rate.

- ▶ Perfectly competitive *firms*  $j \in [0, 1]$ 
  - ▶ produce intermediate goods  $y_{j,t}^m = a_t f(n_{j,t} k_{j,t-1})$ , where  $a_t$  is stochastic
  - ▶ accumulates physical capital with investment adjustment costs  $\Gamma_I(x_{j,t}/x_{j,t-1})$
- ▶ Wages have to be paid up-front such that firms demand loans  $L_{j,t}$  from banks

$$L_{j,t}/R_t^L \geq P_t w_t n_{j,t}. \quad (4)$$

Liquidity constraint (4) distorts labor demand if  $R_t^L / R_t^{Euler} > 1$ :

$$Z_t a_t f_n(n_{j,t} k_{j,t-1}) = P_t w_t \cdot \left( R_t^L / R_t^{Euler} \right).$$

- ▶ Monopolistically competitive *retailers* buy intermediate goods at price  $Z_t$ 
  - ▶ Retailer  $k \in [0, 1]$  relabels the intermediate good to  $y_{k,t}$  and
  - ▶ Retailer set prices in a sticky way (a'la Calvo)
  - ▶ They sell  $y_{k,t}$  at  $P_{k,t}$  to perfectly competitive *bundlers* (who bundle the  $y'_{k,t}$ s to the final good  $y_t$ )
- ▶ Standard New Keynesian welfare costs of price dispersion (and thus inflation)

- ▶ The government issues nominal long-term debt as perpetuities with coupon payments that decay exponentially at the rate  $\rho \in [0, 1]$ .
- ▶ raises lump-sum taxes  $\tau_t$  and purchases goods  $g_t$
- ▶ The flow budget constraint of a government can be written as

$$p_t^L B_t^T + P_t s p_t = (1 + \rho p_t^L) B_{t-1}^T, \quad \text{with } p_0^B B_{-1}^T > 0, \quad (5)$$

- ▶ Government is perfectly committed to pay the coupon  $\rho$  in all periods and states
- ▶ government controls the primary surplus according to the following feedback rule.

$$P_t s p_t = \gamma_b \cdot (1 + \rho p_t^L) B_{t-1}^T + \gamma_y \cdot P_t y_t + \varepsilon_t^{sp}, \quad \gamma_{y,b} \geq 0, \quad (6)$$

- ▶ government spending  $\{g_t\}_{t=0}^{\infty}$ , which is assumed to evolve according to

$$g_t = \rho_g g_{t-1} + (1 - \rho_g) g + \rho_{gy} y_{t-1} + \varepsilon_{g,t},$$

# Calibrated parameter

parameter	value	description
household preferences		
$\upsilon$	1	Frisch labor supply elasticity
$\beta$	0.984	time discount
$\varrho$	0.01	scale parameter for utility of deposits
technology		
$\delta$	0.03	depreciation rate
$\alpha$	0.75	labor share
price and wage setting		
$\epsilon_{\pi}$	6.00	mark-up prices
$\epsilon_w$	6.00	mark-up wages
intermediation and policy		
$\mu$	0.025	reserve policy
$\lambda$	0.1	fraction of money held outright
$\bar{\kappa}$	1	money supply control
steady state values		
$\bar{n}$	1/3	labor supply
$\bar{\pi}$	1.0108	inflation
$R_m$	1.0159	interest rate



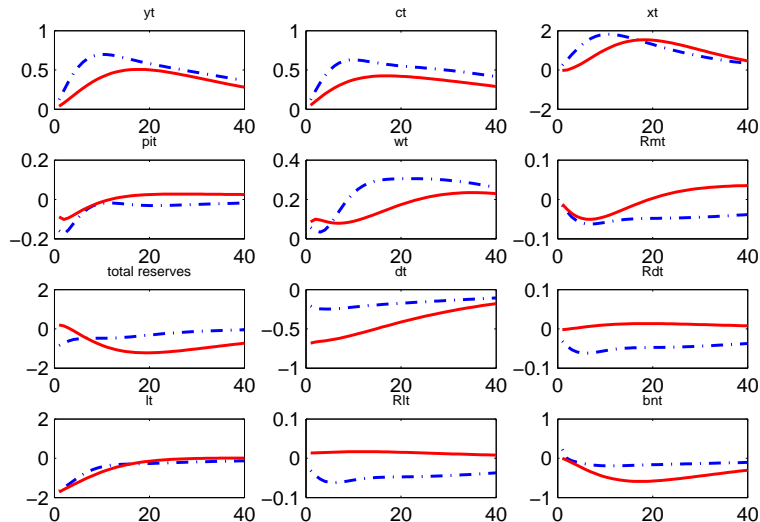
# Estimation results

Parameter		Prior			Posterior			
		Type	Mean	Std	Mode	Mean	5% CI	95% CI
<b>FIRMS AND HOUSEHOLDS</b>								
Calvo wages	$\phi_p$	$\mathcal{B}$	0.700	0.1500	0.8090	0.8119	0.7756	0.8489
Calvo prices	$\phi_w$	$\mathcal{B}$	0.700	0.1500	0.8888	0.8780	0.8183	0.9372
reci. of intertemporal elasticity	$\sigma$	$\mathcal{G}$	1.000	0.5000	2.1707	2.4948	1.5631	3.3628
investment adjustment	$\gamma_i$	$\mathcal{G}$	6.000	2.5000	7.1527	9.4263	5.4172	13.2959
<b>BANKING</b>								
credit cost 1	$\bar{\zeta}$	$\mathcal{G}$	1.000	0.5000	0.4763	0.5381	0.1507	0.9508
credit cost 2	$\eta^{rc}$	$\mathcal{G}$	0.010	0.0050	0.0051	0.0055	0.0013	0.0095
credit cost 3	$\omega$	$\mathcal{G}$	2.500	0.5000	3.4954	3.5139	2.6458	4.3921
<b>INTEREST RATE RULE</b>								
Interest rate smoothing	$\rho_r$	$\mathcal{B}$	0.700	0.1000	0.9001	0.8998	0.8671	0.9333
Resp. to inflation	$\phi_\pi$	$\mathcal{G}$	1.500	0.2000	1.3922	1.4240	1.1080	1.7352
Resp. to output	$\phi_y$	$\mathcal{G}$	0.010	0.0010	0.0099	0.0100	0.0084	0.0115
<b>SHOCK PERSISTENCE</b>								
Mark-up shock prices	$\rho_{\epsilon_p}$	$\mathcal{B}$	0.700	0.1500	0.9467	0.9368	0.8974	0.9778
Mark-up shock wages	$\rho_{\epsilon_w}$	$\mathcal{B}$	0.700	0.1500	0.6575	0.6559	0.4755	0.8447
Banking cost shock	$\rho_{\eta_{rc}}$	$\mathcal{B}$	0.700	0.1500	0.9264	0.9239	0.9003	0.9474
Collateral shock (OMO)	$\rho_{OMO}$	$\mathcal{B}$	0.700	0.1500	0.9111	0.8809	0.8032	0.9612
Preference shock	$\rho_\xi$	$\mathcal{B}$	0.700	0.1500	0.8315	0.8092	0.7215	0.8995
Technology shock	$\rho_{tfp}$	$\mathcal{B}$	0.700	0.1500	0.9606	0.9567	0.9322	0.9809
Investment shock	$\rho_x$	$\mathcal{G}^{-1}$	0.700	0.1500	0.3197	0.3433	0.1705	0.5096
Government spending shock	$\rho_g$	$\mathcal{G}^{-1}$	0.700	0.1500	0.8801	0.8728	0.7856	0.9696
<b>STANDARD DEVIATIONS</b>								
Preference shock	$\sigma_\xi$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0403	0.0465	0.0301	0.0617
Technology shock	$\sigma_{tfp}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0140	0.0143	0.0125	0.0160
Interest Rate shock	$\sigma_{r_m}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0784	0.0817	0.0653	0.0981
Mark-up shock prices	$\sigma_{\epsilon_p}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.4426	0.4927	0.3161	0.6626
Mark-up shock wages	$\sigma_{\epsilon_w}$	$\mathcal{G}^{-1}$	0.050	0.5000	1.1503	1.3409	0.2242	2.8121
Investment shock	$\sigma_{\epsilon_x}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0706	0.0924	0.0493	0.1326
Banking cost shock	$\sigma_{\eta_{rc}}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0299	0.0308	0.0260	0.0354
Collateral shock (OMO)	$\sigma_{OMO}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0077	0.0080	0.0065	0.0094
Government spending shock	$\sigma_{OMO}$	$\mathcal{G}^{-1}$	0.050	0.5000	0.0162	0.0164	0.0144	0.0183

## (Almost) conventional policy rate effects

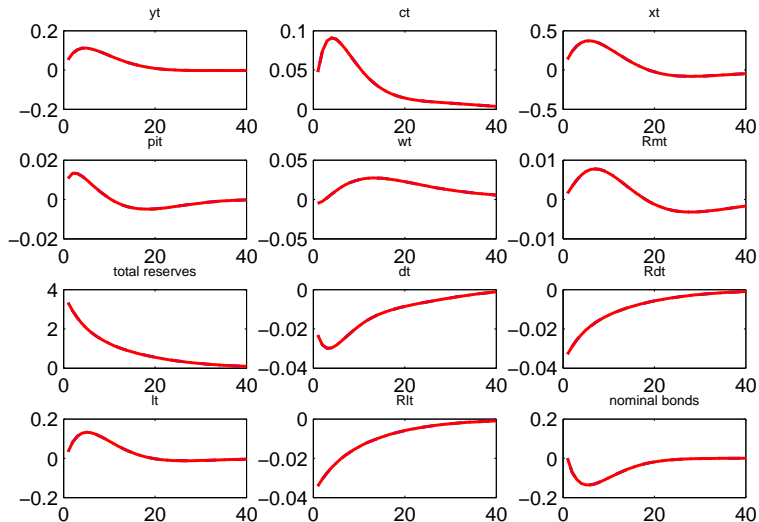
- ▶ The model exhibits policy rate effects consistent with empirical (VAR) evidence
  - ▶ Increase in  $R_t^m$  reduces real activity and inflation
  - ▶ Value of government bonds increase due to an increased real rate
- ▶ Policy rate effects can be altered by additional instruments
  - ▶ Neutralized money supply and fraction of outright purchases (20%↑)

# TFP shock



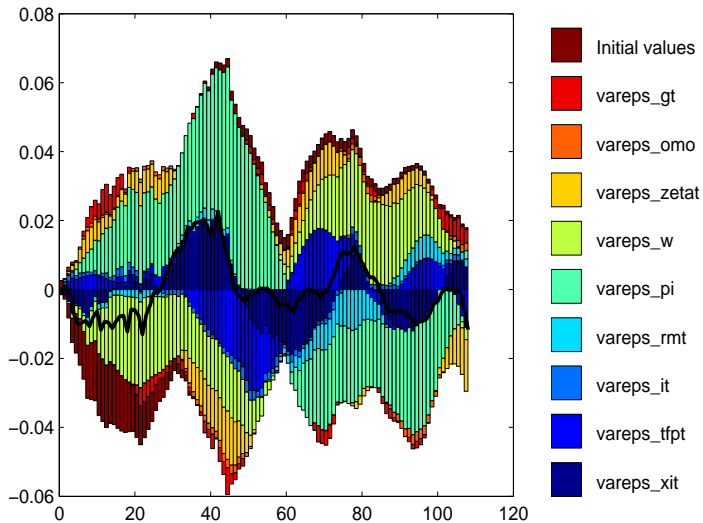
Impulse responses to a TFP shock under binding collateral constraint (red solid line), the broken blue line gives the responses under the assumption of a slack collateral constraint.

# Collateral shock



Impulse responses to a collateral shock under binding collateral constraint (red solid line)

# Observed variable decomposition



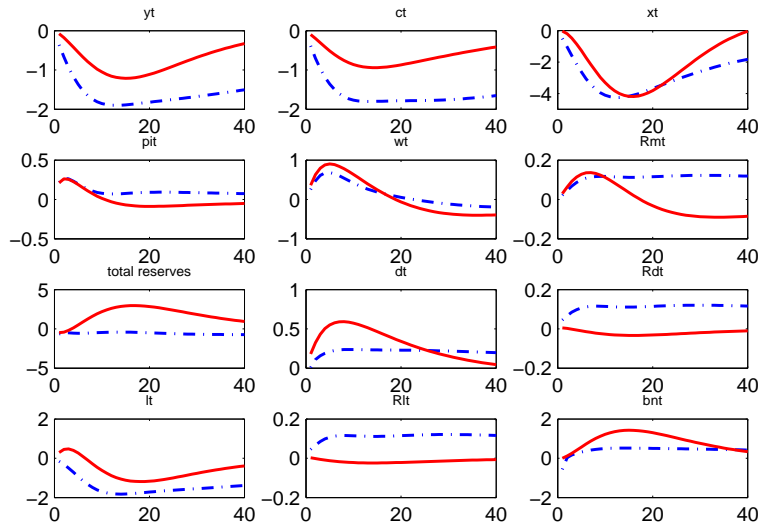
# Variance decomposition (4 quarters)

Forecast horizon: 4 quarters									
Variable	Shock Contribution								
	$\varepsilon_{\xi}$	$\varepsilon_{tfp}$	$\varepsilon_x$	$\varepsilon_{r^m}$	$\varepsilon_{\pi}$	$\varepsilon_w$	$\varepsilon_{\zeta}$	$\varepsilon_{OMO}$	$\varepsilon_g$
output	50.32	2.18	3.33	3.42	12.60	12.07	5.73	1.32	9.0
inflation	3.29	7.67	0.37	1.72	33.84	52.38	0.49	0.14	0.1
consumption	75.16	2.60	0.10	2.03	4.47	11.07	3.59	0.83	0.1
investment	0.50	0.29	33.62	5.76	42.36	6.70	8.76	1.97	0.0
loans	15.77	62.23	0.58	0.97	12.25	4.42	0.96	0.24	2.5
employment	15.80	72.93	0.36	1.09	2.56	2.40	1.71	0.40	2.7
wages	0.04	1.34	0.22	0.04	23.22	75.10	0.02	0.00	0.0
reserves	13.40	0.18	0.57	1.11	1.06	1.16	1.50	78.40	2.6
deposits	1.28	72.60	0.48	0.52	0.33	23.70	0.62	0.13	0.3
nominal bond	47.94	2.11	3.92	5.13	11.72	11.48	5.12	1.55	11.0
lending rate	1.15	2.79	0.05	0.14	0.29	0.45	82.60	12.35	0.1
policy rate	3.02	6.25	0.24	19.88	28.31	41.63	0.43	0.12	0.1
deposit rate	6.92	0.08	0.27	0.63	0.62	0.42	67.95	22.01	1.1

# Variance decomposition (40 quarters)

Forecast horizon: 40 quarters									
Variable	Shock Contribution								
	$\varepsilon_{\xi}$	$\varepsilon_{tfp}$	$\varepsilon_x$	$\varepsilon_{r^m}$	$\varepsilon_{\pi}$	$\varepsilon_w$	$\varepsilon_{\zeta}$	$\varepsilon_{OMO}$	$\varepsilon_g$
output	5.24	12.13	0.57	0.77	24.38	55.19	0.98	0.20	0.55
inflation	3.69	7.56	0.65	2.21	28.70	56.41	0.54	0.14	0.11
consumption	11.68	13.69	0.72	0.66	17.45	54.60	0.89	0.18	0.13
investment	3.26	7.94	1.16	0.92	36.69	48.66	1.09	0.22	0.07
loans	5.34	17.42	0.23	0.83	47.77	27.09	0.66	0.14	0.52
employment	6.36	32.53	0.09	0.88	15.93	42.08	1.11	0.23	0.77
wages	0.78	4.60	0.59	0.31	60.10	33.22	0.32	0.05	0.03
reserves	4.71	9.85	0.43	0.83	20.57	46.62	0.86	15.71	0.43
deposits	0.65	58.86	0.44	0.17	0.57	38.82	0.33	0.06	0.08
nominal bond	4.68	12.02	0.60	0.85	24.83	55.34	0.94	0.22	0.51
lending rate	0.97	9.78	0.14	0.18	2.11	14.45	63.98	8.32	0.08
policy rate	4.06	7.79	0.73	3.46	25.20	57.97	0.55	0.13	0.11
deposit rate	3.37	6.88	0.30	0.59	14.83	33.18	31.56	8.98	0.31

# Wage mark-up shock



Impulse responses to a wage mark-up shock under binding collateral constraint (red solid line), the blue broken line gives the responses under the assumption of a slack collateral constraint.



# RE equilibrium (1)

## Definition

A RE equilibrium under risk-free public debt is given by a set of sequences  $\{c_t, \lambda_t, n_t, d_t, \pi_t, w_t, mc_t, k_t, x_t, q_t, \eta_t, m_t, m_t^R, pb_t, pb_t^T, l_t, i_t, \tilde{Z}_t, y_t, s_t, R_t^L, R_t^d, R_t^b, R_t^{Euler}, \varphi_{t,t+1}, f_t^1, f_t^2, p_t^B, b_t^T, g_t, \tau_t\}_{t=0}^{\infty}$  satisfying

$$\xi_t u_{c,t} = \lambda_t, \quad (7)$$

$$1/R_t^d = E_t \left[ \varphi_{t,t+1} \left( 1 + \frac{u_{d,t+1}}{u_{c,t+1}} \right) \right], \quad (8)$$

$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}}, \quad (9)$$

$$1/R_t^{Euler} = E_t \varphi_{t,t+1}, \quad (10)$$

$$mc_t \alpha a_t n_t^{\alpha-1} k_{t-1}^{1-\alpha} = \mu_t^m \cdot w_t \cdot \left( R_t^L / R_t^{Euler} \right), \quad (11)$$

$$l_t / R_t^L = w_t n_t, \quad (12)$$

## RE equilibrium (2)

$$w_t = [\varsigma w_{t-1}^{1-\varepsilon_w} \left(\frac{\pi_t}{\pi_{t-1}}\right)^{\varepsilon_w-1} + (1-\varsigma)\tilde{w}_t^{1-\varepsilon_w}]^{1/(1-\varepsilon_w)}, \quad (13)$$

$$f_t^1 = f_t^2, \quad (14)$$

$$f_t^1 = \tilde{w}_t \xi_t u_{c,t} (w_t/\tilde{w}_t)^{\varepsilon_w} n_t + E_t \beta \varsigma \left(\frac{\pi_{t+1}}{\pi_t}\right)^{\varepsilon_w-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\varepsilon_w-1} f_{t+1}^1, \quad (15)$$

$$f_t^2 = \nu_t \xi_t \frac{\varepsilon_w}{\varepsilon_w - 1} \left(\frac{w_t}{\tilde{w}_t}\right)^{(1+\nu)\varepsilon_w} n_t^{(1+\nu)} + \beta \varsigma \left(\frac{\pi_{t+1}}{\pi_t}\right)^{(1+\nu)\varepsilon_w} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{(1+\nu)} \quad (16)$$

## RE equilibrium (3)

$$k_t = (1 - \delta)k_{t-1} + \epsilon_t^I \left( 1 - \frac{\gamma_I}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right) x_t \quad (1)$$

$$1 = q_t \epsilon_t^I \left( 1 - \frac{\gamma_I}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 - \gamma_I \left( \frac{x_t}{x_{t-1}} - 1 \right) \frac{x_t}{x_{t-1}} \right) \quad (1)$$

$$+ \beta E_t \left[ \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} q_{t+1} \epsilon_{t+1}^I \gamma_I \left( \frac{x_{t+1}}{x_t} - 1 \right) \left( \frac{x_{t+1}}{x_t} \right)^2 \right]$$

$$q_t = \beta E_t \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left[ q_{t+1} (1 - \delta) + (m c_{t+1} / \mu_{t+1}^m) (1 - \alpha) a_{t+1} n_{t+1}^\alpha k_t^{-\theta} \right]$$

# RE equilibrium (4)

$$\frac{1}{R_t^d} = 1 + (1 - \mu)E_t\varphi_{t,t+1}\Xi_{m,t+1} \quad (20)$$

$$\frac{1}{E_t R_{t+1}^B} = \frac{1}{R_t^d} + \mu E_t \varphi_{t,t+1} \Xi_{m,t+1} + \frac{E_t R_{t+1}^B \varphi_{t,t+1} \eta_{t+1} \kappa_{t+1}}{E_t R_{t+1}^B} \quad (21)$$

$$\frac{1}{R_t^L} = \frac{1}{R_t^d} + \mu E_t \varphi_{t,t+1} \Xi_{m,t+1} - \Xi_{l,t}, \quad (22)$$

$$R_t^m \eta_t = -(R_t^m - 1) - \Xi_{m,t}, \quad (23)$$

$$d_t = m_t + E_t p_{t+1}^B b_t + l_t \quad (24)$$

$$i_t \leq \kappa_t (p b_t + \epsilon_{t,omo}) \pi_t^{-1} / R_t^m, \quad (25)$$

$$i_t = m_t - m_{t-1} \pi_t^{-1} + m_t^R \quad (26)$$

$$m_t = \Lambda m_t^R, \quad (27)$$

$$p b_t = p b_t^T - m_{t-1}, \quad (28)$$

$$p b_t^T = p_t^B b_{t-1}^T, \quad (29)$$

$$R_t^b = \frac{\rho p_t^B}{p_{t-1}^B - 1}, \quad (30)$$

## RE equilibrium (5)

$$y_t = a_t n_t^\alpha k_{t-1}^{1-\alpha} / s_t, \quad (33)$$

$$y_t = c_t + x_t + g_t + \Xi_t, \quad (34)$$

$$s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \left( \frac{\pi_t}{\pi_{t-1}^L} \right)^\varepsilon, \quad (35)$$

(where  $1/R_t^{Euler} = E_t \varphi_{t,t+1}$ ,  $u_{ct} = [c_t - hc_{t-1}]^{-\sigma}$ ,  $u_{dt} = \varrho d_t^{-\varphi}$ ,  $u_{nt} = -\nu_t n_t^\nu$ ,  $\tilde{R}_t^b = R_t^b \pi_t^{-1}$ ,  $R_{t+1}^b = p_{t+1}^B / q_t^B$ ,

$\Xi_t(l_t, i_t) = \zeta_t \left( \frac{l_t}{(m_{t-1} \pi_t^{-1} - \mu d_{t-1} \pi_t^{-1} + i)^\omega} \right)^{\eta_{rc}}$ ,  $\Xi_{l,t} = \eta_{rc} \Xi_t / l_t$  and

$\Xi_{m,t}(l_t, i_t) = -\eta_{rc} \Xi_t (m_{t-1} \pi_t^{-1} - \mu d_{t-1} \pi_t^{-1} + i)^{-1}$ , as well as the transversality conditions, a monetary policy setting  $\{R_t^m \geq 1\}_{t=0}^\infty$  and  $\pi \geq \beta$ , and  $\kappa_t$

and a fiscal policy satisfying

$$\frac{p_t^B - 1}{\rho} b_t^T + \tau_t - g_t = p_t^B b_{t-1}^T / \pi_t, \quad (36)$$

$$\tau_t - \tau = g_t - g + \rho_{\tau b} \cdot [(1 + \rho p_t^L) b_{t-1}^T \pi_t^{-1} - (1 + \rho p^L) b^T \pi^{-1}] + \rho_{\tau y}$$

$$g_t - g = \rho_g (g_{t-1} - g) + \rho_{gy} (y_{t-1} - y) + \varepsilon_{gt},$$

for stochastic processes  $\{a_t, \xi_t, \varepsilon_{r,t}, \varepsilon_t, \eta_t, \zeta_t, \epsilon_{t,omo}\}_{t=0}^{\infty}$  and given initial values  $m_{-1} > 0$ ,  $l_{-1} > 0$ ,  $pb_{-1}^T > 0$ ,  $pb_{-1} > 0$ ,  $k_{-1} > 0$ ,  $x_{-1} > 0$ ,  $\pi_{-1} > 0$ , and  $s_{-1} \geq 1$ .