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FINANCING CONSTRAINTS, FIRM DYNAMICS AND INNOVATION

Andrea Caggese

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Abstract

This paper develops the model of an industry with heterogeneous firms, and studies the effect of financing frictions and bankruptcy risk on innovation and aggregate productivity growth. The model has two main features: i) the technology of firms gradually becomes obsolete. Firms can counter this process by innovating, but the innovation outcome is risky. ii) Financial frictions cause the inefficient default of financially fragile firms, deter entry, and reduce competitive forces in the industry. I calibrate and solve the model and simulate several industries, and show that financing frictions have two distinct effects on innovation: a "direct effect", for firms that cannot innovate because of lack internal funds to invest, and an "indirect effect", where the changes in competition and profitability change also the incentives to innovate. Simulation results first show that, for realistic parameter values, the indirect effect of financing frictions is much more important than the direct effect in determining the innovation decisions. Second, they show that "Safe innovation" (where firms invest to upgrade their technology and are certain to increase their productivity) is increased by the presence of financing frictions, because the reduction in competition increases the return on innovation. Conversely "Risky innovation" (where firms invest to improve their productivity, but with some probability fail to do so and end up reducing their productivity instead), is discouraged by financing frictions. This happens because the reduction in competition implies that firms remain profitable for a longer time and therefore they wait longer before attempting a risky innovation process. I test these predictions and their implications for productivity growth on a sample of Italian manufacturing firms, and I find that the life cycle and innovation decisions of firms are fully consistent with the model with risky innovation.


1 Introduction

A large body of literature on firm dynamics has emphasized that firms and plants are created small and tend to grow over time (see for example Davis, Haltiwanger, and Schuh, 1996). The process of entry, growth and exit is largely driven by technological progress. New entrants adopt technologies at the frontier and are more productive than existing units. Moreover firms and plants that successfully innovate expand, while unsuccessful ones shrink and disappear. This process of selection, innovation and growth at the firm level is an important factor in driving aggregate productivity growth. A recent paper by Hsieh and Klenow (2012) compares the life cycle of manufacturing plants in the USA, Mexico and India. Hsieh and Klenow (2012) show that the average size and total factor productivity of surviving plants strongly increases along the plants life cycle in the USA, while such positive relation is much more tenuous in Mexico and absent in India. Among the possible factors that could explain these differences in the life cycle of plants in different economies is the presence of financial market imperfections.

This paper investigates on this possibility by studying a model with financing frictions, bankruptcy risk, and innovation decisions of firms, and derives quantitative implications for the lifecycle of firms and for aggregate productivity. The model considers firm dynamics a la Hopenhayn (1992), with three main features: i) Firms are monopolistically competitive. Entry costs determine the number of entry and exit, the number of firms operating in equilibrium and their expected profits; ii) Firms face financing frictions. They are hit by idiosyncratic revenue shocks that causes losses, reduce their net financial wealth, and may cause inefficient liquidation; iii) New firms are able to access a better and more productive technology than the average incumbent firm. But as firms age, their technology becomes obsolete with some probability, and
their productivity falls relative to the new entrants. However firms can counter the obsolescence of their product by innovating. Innovation is costly, and also potentially risky. If it succeeds, it improves productivity and revenues (i.e. the new product is a success). If it fails, it reduces them (i.e. the new product is a failure).

I solve the model and calibrate an industry to match the firm dynamics of a sample of Italian manufacturing firms. This sample comprises a rich dataset with balance sheet information for the 1989-2000 period as well as detailed survey information on innovation and financing frictions at the firm level. I calibrate a benchmark industry with an average level of financial frictions, as well as two alternative industries, one with low financing frictions, and one with high financing frictions. In this model financing frictions have two distinct effects on innovation: a "direct effect", for firms that cannot innovate because they lack internal funds to invest, and an "indirect effect", because they change the incentives to innovate by altering the level of competition and profitability. This second effect happens because, as is shown in Caggese and Cunat (2012), financing frictions increase the chances that firms go bankrupt in their early stages of life. This discourages entry and reduces competition for firms that survive and accumulate sufficient financial assets to self insure against the bankruptcy risk.

In order to understand the implication of these two effects for innovation and productivity over the life cycle, I simulate several industries for many periods and for two different types of innovation. In the first case innovation is relatively "safe", in the sense that while the precise outcome of innovation may not be certain, it always improves productivity with respect to not innovating. In the second case innovation is "risky", in the sense that with some probability innovation fails and productivity falls more than it would have done without innovating.

Simulation results highlight two main findings: first, for realistic parameter values the indirect effect of financing frictions is much more important than the direct effect
in determining the innovation decisions. While just a small fraction of firms is unable to innovate because of a binding financing constraint, all firms in the industry are significantly affected by the changes in competition induced by financing frictions.

Second, the indirect effect of financing frictions depends on the type of innovation. When innovation is "safe", firms compare the innovation cost today with the expected increase in future productivity and profits. Financing frictions increase this type of innovation because they reduce competition and increase expected profit conditional on all levels of productivity, and especially at the frontier.

Conversely, when innovation is risky, firms also take into account the downside risk: if innovation fails, it reduces considerably the current level of profits. In this case, since financing frictions imply that surviving firms remain profitable for a longer time conditional on not innovating, they also imply that these firms wait longer before risking to innovate relative to an industry with less financing frictions and more competition.

In the second part of the paper I test these predictions, and I find that the life cycle and innovation decisions of firms in the empirical dataset are consistent with those in the simulated industry with risky innovation. First, I show that in more financially constrained sectors firms innovate less and their productivity grows less over time than in less financially constrained sectors. Second, I show that in more financially constrained sectors firms engage relatively more in R&D directed to improve current products and current productive processes, which is a type of less risky innovation, while in less financially constrained sectors firms engage relatively more in R&D directed to introduce new products, which is riskier type of innovation.

2 Related literature

To be added
3 Model

I consider an industry model of firm dynamics a la Hopenhayn (1992), with profits uncertainty and financing frictions as in Caggese and Cunat (2013). The novelty of this model is to introduce technological progress by new entrants and endogenous innovation decisions of incumbents.

In the model firms pay a fixed cost to enter the industry and a fixed cost to operate each period. They are subject to financing frictions when their wealth is low, and they can increase their productivity by engaging in (possibly risky) innovation activity. More formally, each firm in an industry produces a variety \( w \) of a consumption good. There is a continuum of varieties \( w \in \Omega \). Consumers preferences for the varieties in the industry are C.E.S. with elasticity \( \sigma > 1 \). The C.E.S. price index \( P_t \) is then equal to:

\[
P_t = \left[ \int p_t(w)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{1}
\]

And the associated quantity of the aggregated differentiated good \( Q_t \) is:

\[
Q_t = \left[ \int q_t(w)^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}
\]

where \( p_t(w) \) and \( q_t(w) \) are the price and quantity consumed of the individual varieties \( w \), respectively. The overall demand for the differentiated good \( Q_t \) is generated by:

\[
Q_t = AP_t^{1-\eta} \tag{3}
\]

where \( A \) is an exogenous demand parameter and \( \eta < \sigma \) is the industry price elasticity of demand. From (2) and (3) the demand for an individual variety \( w \) is:

\[
q_t(w) = A \frac{P_t^{\sigma-\eta}}{p_t(w)^{\sigma}} \tag{4}
\]

Each variety is produced by a firm using labour, which is paid a unitary wage equal to
1. Therefore total labour cost for one firm is equal to:

$$\text{labour cost} = \frac{q_t(w)}{v}$$  \hspace{1cm} (5)

Where $\frac{1}{v}$ is the labour cost to produce one unit of output. I assume that the marginal productivity of labour for the frontier technology is equal to $v^\infty$, and it grows every period at the gross rate $g > 1$. To normalize the model, I assume that labour cost also grows at the same rate and is equal to $v^\infty$. It is then straightforward to notice that $v$ in equation (5) is equal to $\frac{\nu}{\nu^\infty}$, the productivity of the firm relative to the frontier technology. The profits for a firm with productivity $\nu$, and variety $w$ are given by:

$$\pi_t(\nu_t, \varepsilon_t) = p_t(w)q_t(w) - \frac{q_t(w)}{v_t} - F_t$$  \hspace{1cm} (6)

Since all the formulas are identical for all varieties, we drop the indicator $w$ from now on. $F_t > 0$ are the overhead fixed costs of production that have to be paid every period. They are subject to an idiosyncratic shock $\varepsilon_t$ which is uncorrelated across firms and possibly correlated over time for each firm:

$$F_t = F + \varepsilon_t$$

Firms are heterogeneous in terms of productivity $\nu_t$. The shock $\varepsilon_t$ introduces uncertainty in profits, and it plays an important role in the presence of financing frictions. By affecting the accumulation of wealth and the the probability of default, the shock also affects both the entry decision, the exit decision, and the equilibrium productivity in the industry. The idiosyncratic shock enters additively in $\pi_t(\nu_t, \varepsilon_t)$ so that it does not affect the firm decision on the optimal price $p_t$ and quantity produced $q_t$. This makes the model both easier to solve and more comparable to the basic model without financing frictions.\textsuperscript{1}

\textsuperscript{1}A multiplicative shock of the type $\varepsilon_t p_t(w)q_t(w)$ would not change the qualitative results of the model, but it would have two main consequences. First, it would imply that the optimal quantity produced $q_t(w)$ would be a function of the intensity of financing frictions, thus making the solution of the problem more complicated. Second, it would imply that expected profits are a function of the volatility of the shock $\varepsilon_t$. 

6
The firm is risk neutral and chooses $p_t$ in order to maximize $\pi_t(v_t, \varepsilon_t)$. The first order condition yields the standard pricing function:

$$p_t = \frac{\sigma}{\sigma - 1} v_t \quad (7)$$

It then follows that:

$$\pi_t(v_t, \varepsilon_t) = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^\sigma} AP^{\sigma - 1} \sigma^\sigma - F_t$$

The timing of the model for a firm which was already in operation in period $t - 1$ is the following. At the beginning of period $t$ with probability $\delta$ its technology becomes useless forever, and the firm liquidates all its assets and stops activity. With probability $1 - \delta$ the firm is able to continue with the current technology. It observes the realization of the shocks $\varepsilon_t$, which determines $\pi_t(v_t, \varepsilon_t)$ and realizes financial wealth $a_t$:

$$a_t = R (a_{t-1} - I_{t-1}K - d_{t-1}) + \pi_t(v_t, \varepsilon_t) \quad (8)$$

$I_{t-1}$ is an indicator function that is equal to 1 if the firm decided to innovate in period $t - 1$. $K$ is the cost of innovation. Financing frictions are introduced by following Caggese and Cuñat (2012) and assuming that the firm cannot borrow to finance the fixed cost of its operations. While it can pay workers with the stream of revenues generated by their labour input, it has to pay in advance for a fraction $\gamma$ of the other costs of production. Therefore continuation is feasible only if:

$$a_t \geq \gamma F \quad (9)$$

where $\gamma$ is a coefficient greater than zero and proxies for the efficiency of the financial system. It is natural to assume that $\gamma$ is $\leq 1$, but it should be possible to argue that $\gamma > 1$ is simply a shortcut to more severe levels of financing frictions. If the constraint (9) is not satisfied, then the firm cannot continue its activity and is forced to liquidate.\(^2\)

\(^2\)Constraint (9) is a simple way to introduce financing frictions in the model. Nonetheless it generates realistic firm dynamics, and can be interpreted as a shortcut for more realistic models of firm dynamics with financing frictions, such as, for instance, Clementi and Hopenhayn (2006).
Conditional on continuation, innovation is feasible if:

\[ a_t \geq \gamma (F + K) \] (10)

Given the presence of financing frictions, and the fact that the firm discounts future profits at the constant interest rate \( R \), it is trivial to show that it is never optimal for the firm to distribute dividends while in operation, since accumulating wealth reduces future expected financing constraints. Hence dividends \( d_t \) are always equal to zero. Profits increase wealth \( a_t \), which is distributed as dividends when the firm is liquidated.

After observing \( \varepsilon_t \) and realizing profits, the firm decides whether or not to continue activity the next period. It may decide not to do so if it is not profitable enough to cover the fixed per period cost \( F \). In this case the firm voluntarily liquidates and ceases to operate forever. Conditional on continuation, it decides whether or not to innovate in period \( t \) to improve productivity in period \( t + 1 \).

If the firm does not innovate \( (I_t = 0) \), it uses its fixed investment \( F \) to improve the current technology. The improvement is successful with probability \( \xi^{NI} \) and increases productivity at the rate \( g > 0 \). Therefore relative productivity \( v \) remains constant. With probability \( 1 - \xi^{NI} \) the improvement is not successful and \( v \) decreases at the rate \( g \). This stochastic incremental innovation is not essential for the qualitative results of the model, but it is necessary to properly calibrate it with empirical data.

If the firm does innovate \( (I_t = 1) \), it will reach the frontier technology \( (v = 1) \) with probability \( \xi^I \), while it will depreciate at the rate \( g^{fail} \) with probability \( 1 - \xi^I \). \( g^{fail} \) is not necessarily equal to \( g \), and it depends on the type of innovation, as explained in section 4.1. The following table summarizes the law of motion of productivity conditional on \( I_t \):
if $I_t = 0$: \[ v_{t+1} = v_t \text{ w.p. } \xi^{NI} \]
\[ v_{t+1} = \frac{a_t}{g} \text{ w.p. } 1 - \xi^{NI} \]

if $I_t = 1$: \[ v_{t+1} = 1 \text{ w.p. } \xi^I \]
\[ v_{t+1} = \frac{a_t}{g^{\gamma t}} \text{ w.p. } 1 - \xi^I \]

In order to characterize the innovation decision, we define $V_t^{UP}(a_t, \varepsilon_t, v_t)$ as the value function today conditional on innovating:

\[
V_t^{UP}(a_t, \varepsilon_t, v_t) = -K_t + \frac{1 - \delta}{R} \left\{ \frac{\xi^I E_t[V_{t+1}(a_{t+1}, \varepsilon_{t+1}, 1) + \pi_{t+1}(\varepsilon_{t+1}, 1)]}{E_t[V_{t+1}(a_{t+1}, \varepsilon_{t+1}, \frac{a_t}{g^{\gamma t}}) + \pi_{t+1}(\varepsilon_{t+1}, \frac{a_t}{g^{\gamma t}})]} \right\}
\]

Likewise, the value function conditional on not upgrading is:

\[
V_t^{NOUP}(a_t, \varepsilon_t, v_t) = \frac{1 - \delta}{R} \left\{ \frac{\xi^{NI} E_t[V_{t+1}(a_{t+1}, \varepsilon_{t+1}, 1) + \pi_{t+1}(\varepsilon_{t+1}, v_t)]}{E_t[V_{t+1}(a_{t+1}, \varepsilon_{t+1}, \frac{a_t}{g^{\gamma t}}) + \pi_{t+1}(\varepsilon_{t+1}, \frac{a_t}{g^{\gamma t}})]} \right\}
\]

The firm then innovates and chooses $I_t(a_t, \varepsilon_t, v_t) = 1$ if condition (10) and the following condition (11) are satisfied:

\[
V_t^{UP}(a_t, \varepsilon_t, v_t) > V_t^{NOUP}(a_t, \varepsilon_t, v_t)
\]

While $I_t(a_t, \varepsilon_t, v_t) = 0$ otherwise. Given the innovation decision, the value of the firm at time $t$, $V_t(a_t, \varepsilon_t, v_t)$, is:

\[
V_t(a_t, \varepsilon_t, v_t) = 1(a_t \geq \gamma F) \left\{ \max \left[ V_t^{UP}(a_t, \varepsilon_t, v_t), V_t^{NOUP}(a_t, \varepsilon_t, v_t), 0 \right] \right\}
\]

Equation 12 implies that the value of the firm is equal to zero in two cases. First, when the indicator function $1(a_t \geq \gamma F)$, which is equal to one if the argument is true, is equal to zero because the liquidity constraint (9) is not satisfied. Second, when the firm is not profitable enough, and the value in the curly brackets is equal to zero.

The technological assumptions described above clarify that, once the value of the firm reaches zero, it stays at zero forever, and the firm never resumes production.

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3I define the value of the firm as the net present value of future profits. Since the discount factor of the firm is $1/R$, and the firm is risk neutral, this value coincides with the net present value of expected dividends.
3.1 Entry decision

Every period there is free entry. There is a large amount of new potential entrants with a constant endowment of wealth \( a_0 \). They draw their type \( v \) from a uniform distribution with support \([v, 1]\), after having paid an initial cost \( S^C \). Once they learn their type they decide whether or not to start activity. If they start, then draw \( \varepsilon_0 \) from an initial distribution. The free entry condition requires that ex ante the expected value of paying \( S^C \) and constituting a firm, conditional on the expectation of the initial value of the shock \( \varepsilon_0 \), is zero:

\[
\int_{\varepsilon}^{1} \max \{ E^{\o} \left[ V_0 (a_0, v_0, \varepsilon_0) \right], 0 \} f(v_0) dv_0 - S^C = 0
\]  

(13)

3.2 Aggregate equilibrium

In the steady state the aggregate price \( P_t \), the aggregate quantity \( Q_t \), and the distribution of firms over the values of \( v_t, \varepsilon_t \) and \( a_t \) are constant over time. The presence of technological obsolescence and the exogenous exit probability \( \delta \) imply that the age of firms is finite and that the distribution of wealth across firms is non-degenerate. Aggregate price \( P_t \) is set to ensure that the free entry condition (13) is satisfied. The number of firms in equilibrium ensures that \( P_t \) also satisfies the aggregate price equation (1). Aggregation is very simple because all operating firms with productivity \( v \) choose the same price \( p(v) \), as determined by (7).

4 Model’s solution and simulations.

In the next subsections I calibrate the model, solve it, and illustrate firm dynamics in the simulated industries.
4.1 Calibration

The calibrated parameters are chosen to match selected moments with the moments estimated from the panel of Italian manufacturing firms analyzed in section 5. This panel is drawn from the Mediocredito/Capitalia surveys of small and medium manufacturing firms. It is an unbalanced panel with annual balance-sheet data and profit and loss statements from 1989 to 2000, as well as qualitative information from three surveys conducted in 1995, 1998 and 2001. Each survey reports information about the activity of the firms in the three previous years and, in particular, it includes detailed information on financing constraints and innovation. The parameters are illustrated in table 1. The scale parameter $A$ matches aggregate sales in the industry. The rate of exogenous depreciation $\delta$ matches the employment share of exiting firms. The average real interest rate is equal to two percent, which is consistent with the average short-term real interest rates in Italy in the sample period. The fixed per period cost $F$ matches the ratio of fixed overhead costs over labor cost, which in our empirical sample is estimated to be around 30% (Caggese and Cuñat 2013). Regarding the distribution of productivity of new firms, $\pi$ is normalized to be equal to 1, while $v$ is set to match the cross sectional dispersion of profitability across firms.

The set of parameters $S^C, \xi$ and $\rho$ determines the profitability of firms. $S^C$ is the initial entry cost, and it matches the average profitability of firms in the industry. The next two parameters define the profits shock $\varepsilon$. This is assumed to be asymmetric, and equal to $\xi$ with probability $\rho$ and equal to 0 with probability $1 - \rho$. The value of $\rho$ matches the fraction of firms in the empirical sample reporting negative net income, while the value of $\xi$ matches the time series volatility of profits for the empirical firms.

The next set of parameters $g, \text{up\_cost}, \alpha^{\text{not}}$ and $\alpha$ determine the life cycle dynamics of firms. The jointly match: the average yearly decline in profits for a non innovating
firm; the cost of innovation relative to added value; the average age of firms, and
the percentage of innovating firms, measured as firms that do R&D to develop and
introduce new products. The parameter $a_0$, the initial wealth of new firms, affects the
intensity of financing frictions and the probability of bankruptcy. We set a benchmark
value such that 0.5% of firms go bankrupt in the simulations. This corresponds to a
relatively low value for the Italian industry.$^4$ Later on we compare simulation results
across industries with different values of $a_0$. The last three parameters are not matched
to our sample of manufacturing firms. The value of $\sigma$, the elasticity of substitution
between varieties, is equal to 4, in line with Bernard, Eaton, Jensen and Kortum (2003),
who calculate a value of 3.79 using plant level data. The value of $\eta$, the industry price
estasticity of demand, is set equal to 1.5, following Constantini and Melitz (2007).$^5$ The
value of $\gamma$, the fraction of fixed costs that need to be financed with internal finance, is
equal to 1.

Finally, the parameter $g_{\text{fail}}$, together with $\alpha$, determines the riskiness of the inno-
vation decision. I consider two types of innovation:

The first type, called "disembodied innovation", is an R&D process with uncertain
outcome. $K$ is the cost to obtain new knowledge. The firm will verify the outcome of
such process and will apply the new ideas only if they can improve productivity. In
this case $g_{\text{fail}} \leq g$, which implies that productivity $v_{t+1}$ is always weakly higher than
in case of not innovating and not improving current productivity:

\[
\begin{align*}
\text{Innovation succeeds} & \quad (v_{t+1} = 1) > (v_{t+1} = \frac{v_t}{g_{\text{fail}}}) \geq (v_{t+1} = v_t) > \\
\text{Innovation fails} & \quad (v_{t+1} = \frac{v_t}{g_{\text{fail}}}) \geq (v_{t+1} = v_t) > \\
\text{Does not innovate and} & \quad (v_{t+1} = v_t) > \\
\text{improves} & \quad (v_{t+1} = \frac{v_t}{g}) > \\
\text{Does not} & \quad (v_{t+1} = \frac{v_t}{g}) > \\
\text{innovate and} & \quad (v_{t+1} = v_t) > \\
\text{does not improve} & \quad (v_{t+1} = \frac{v_t}{g}) > \\
\end{align*}
\]

$^4$As a comparison, business bankruptcies in Italy in the recent deep 2009-2012 recession have been
around 2-3% per year.

$^5$This is also consistent with Broda and Weinstein (2006), who estimate that the elasticity of sub-
stitution falls between 33% to 67% moving from the highest to the lowest level of disaggregation in
industry data.
The second type is called "embodied innovation". In this case $K$ is the cost of investment in a new type of physical capital in which the new technology is embodied. Such investment is irreversible, and its return depends on the success of the innovation. In case of failure, it is too costly to revert to the old technology. In this case $g^{fail} > g$, and productivity conditional on failure is smaller than in the case of not innovating:

<table>
<thead>
<tr>
<th>Innovation succeeds</th>
<th>Does not innovate and improves</th>
<th>Does not innovate and does not improve</th>
<th>Innovation fails</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(v_{t+1} = 1)$</td>
<td>$(v_{t+1} = v_t)$</td>
<td>$(v_{t+1} = \frac{v_t}{g})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(v_{t+1} = \frac{v_t}{g^{fail}})$</td>
<td></td>
</tr>
</tbody>
</table>

"Embodied innovation" as defined above implicitly assumes that current assets are not suitable to implement the new product, and is therefore similar to the concept of "Radical Innovation", well known in management studies. Utterback (1996) defines Radical Innovation as a "change that sweeps away much of a firm’s existing investment in technical skill and knowledge, designs, production technique, plant and equipment".

For the benchmark calibration I consider the "Embodied innovation" case, and set a benchmark value of $g^{fail}$ such that productivity depreciation after a failed innovation is 3 times larger than depreciation without innovating. This corresponds to a fall in profits by around 10% if innovation fails. Later on I compare the behaviour of several industries with different values of $g^{fail}$.

### 4.2 Simulation results

We simulate the benchmark industry for many periods and we compute the steady state equilibrium. We also simulate two alternative industries, identical to the benchmark one except than for the value of $w_0$. The "Constrained" industry has a low value of $a_0 = 0.21$ and an higher intensity of financing frictions: 1.58% of firms go bankrupt in this industry every period, or approximately 3 times as much as in the benchmark.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Value</th>
<th>Moment to match</th>
<th>Data</th>
<th>Baseline simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Aggregate sales</td>
<td>5610</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>employment share of exiting firms</td>
<td>0.03</td>
<td>8.2%</td>
</tr>
<tr>
<td>$r$</td>
<td>average real interest rate</td>
<td>1.02</td>
<td>2%</td>
</tr>
<tr>
<td>$F$</td>
<td>average ratio fixed costs/labour costs</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
<td>normalized to 1.</td>
<td>1</td>
<td>n.a.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Cross sectional dispersion of firm average profits/added v.</td>
<td>0.969</td>
<td>0.084</td>
</tr>
<tr>
<td>$S^C$</td>
<td>mean profits/added value</td>
<td>0.6</td>
<td>0.084</td>
</tr>
<tr>
<td>$\xi$</td>
<td>average of time series vol of profits/ad.v. at the firm level</td>
<td>0.3</td>
<td>0.092</td>
</tr>
<tr>
<td>$\rho$</td>
<td>fraction of negative profits</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>$g$</td>
<td>average yearly decline in profits/sales. for a non inn. firm</td>
<td>1.0035</td>
<td>3%</td>
</tr>
<tr>
<td>$K$</td>
<td>average r&amp;d/added value</td>
<td>0.02</td>
<td>3%</td>
</tr>
<tr>
<td>$\alpha^{not}$</td>
<td>average age of firms</td>
<td>0.5</td>
<td>24</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>% of innovating firms (r&amp;d to introduce new products)</td>
<td>0.1</td>
<td>13%</td>
</tr>
<tr>
<td>$a_0$</td>
<td>% of firms going bankrupt every period</td>
<td>0.4</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

Other parameters

| $\eta$ | 1.5         |
| $\sigma$ | 4          |
| $\gamma$ | 1        |
| $g^{fail}$ | $g^3$    |

benchmark (sensitivity analysis)
industry. The other alternative industry is the "Unconstrained industry". It has a high value of $a_0 = 2$ and a near zero frequency of bankruptcy.

Figure 1 shows the frequency of firms that innovate conditional on age, for the "Embodied innovation" case. Firms in the "Unconstrained" industry start to innovate earlier in life than firms in the constrained group. Higher competition in this economy lowers profits at all productivity levels. So firms reach sooner the level at which either they "gamble for resurrection and innovate", or they are not profitable any more. The risk of innovation is to become even more unprofitable, but this is the less important the more firms are close to the productivity level at which they exit activity. Conversely in the "Constrained" industry firms are on average more profitable and wait longer before starting to innovate. The implication is that fewer firms innovate in equilibrium, 11.5% against a value of 12.9% in the Unconstrained industry, and firms are on average less productive. Figure 2 shows the life cycle of productivity in the two industries, relative to the benchmark group. Because firms innovate later in the Constrained industry,
average productivity declines from age 23 on relative to the Unconstrained industry.

Figures 3 and 4 repeat the same analysis as figures 1 and 2 but this time considering a calibration with "disembodied" innovation instead, where $g_{\text{fail}} = 1 < g$. In this case a failed innovation does not reduce productivity relative to the previous situation. They show that the results are reversed. Now firms in the constrained industry innovate more, and their productivity grows faster over time relative to the firms in the unconstrained industry.

In order to give some intuition of the relation between innovation risk and innovation in the different industries, figure 5 shows profitability of firms for a given productivity level for the embodied innovation case, starting from 1 (the upper bound of productivity of new firms and the level reached by the firms that successfully innovate) on [this figure has to be redone, it is not clear at all!!]. Profits decline with productivity, and they are uniformly higher in the industry with financing frictions, because of lower competition. These higher profits compensate for the risk of going bankrupt early in
Figure 3: Frequency of firms that innovate conditional on age - SAFE INNOVATION

Figure 4: Average productivity conditional on age, relative to the average productivity conditional on age in the benchmark industry, SAFE INNOVATION.
Figure 5: Profits/sales ratio as a function of productivity, benchmark calibration with risky innovation

With high competition and risky innovation, firms start to innovate here.

With low competition and risky innovation, firms have still too much to lose to start innovating here. But if innovation is not risky, they innovate more because of the higher returns at the frontier.

life. The vertical line indicates the productivity level at which firms start to innovate in the unconstrained industry. At this level profits are low and the cost associated to failing to innovate is small, because the firm can always reduce losses by exiting from production. Conversely at the same level of productivity profits are higher in the industry with financing constraints. Firms are still far from the region in which they exit, and do not want to risk the current profitability levels by engaging in risky innovation.
5 Empirical evidence

In this section we provide some empirical evidence on the age profile of size, innovation and productivity for our sample of Italian manufacturing firms. The relation between age and productivity may be affected by industry characteristics. For example we may find that larger firms are more productive simply because more productive industries are skewed towards older firms. To control for these industry fixed effects, we follow Hsieh and Klenow (2012), and we first calculate the relevant statistic for each age and for each four digit manufacturing industry. Then we calculate for each age the weighted average across all the industries. Therefore all the results presented in this section show the average within industry relation between age and firm characteristics.

Among all the firms in the dataset I select only firms with less than 500 employees, because the sample is only representative of firms between 10 and 500 employees. Moreover I use the survey information about financing frictions to create a binary variable "\textit{constrained}_{i,s}" at the firm-survey level, which is equal to one if the firm $i$ declares to face financing frictions in survey period $s$, and is equal to zero otherwise.\footnote{Firms report whether they had a loan application turned down recently, whether they desire more credit at the market interest rate and whether they would be willing to pay a higher interest rate than the market rate to obtain credit. I aggregate these three variables into a single variable \textit{constrained} that takes value one if the firm answers yes to any of these three questions. According to this measure, 17 percent of the firms declare themselves to be financially constrained.} Then I calculate the frequency of financially constrained firms in each 4 digit manufacturing sector and I select sectors in two groups: one group is composed by the 25\% four digit sectors with most constrained firms, called the "Constrained" group, and the other group is composed of the 25\% four digit sectors with least constrained firms, called the "Unconstrained" group. These groups are the empirical counterpart of the Constrained and Unconstrained industries analyzed in the previous section. Table 2 shows the distribution of firms in the two groups for each two digit manufacturing sector. It
Table 2: Frequency of constrained and unconstrained firms in each 2 digit manufacturing sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>Class.</th>
<th>n. observations</th>
<th>% of firms in the &quot;Constrained&quot; group</th>
<th>% of firms in the &quot;Unconstrained&quot; group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and Drinks</td>
<td>15</td>
<td>960</td>
<td>74%</td>
<td>13%</td>
</tr>
<tr>
<td>Textiles</td>
<td>17</td>
<td>1150</td>
<td>26%</td>
<td>25%</td>
</tr>
<tr>
<td>Shoes and Clothes</td>
<td>18</td>
<td>551</td>
<td>38%</td>
<td>62%</td>
</tr>
<tr>
<td>Wood Furniture</td>
<td>20</td>
<td>343</td>
<td>36%</td>
<td>21%</td>
</tr>
<tr>
<td>Paper</td>
<td>21</td>
<td>379</td>
<td>37%</td>
<td>28%</td>
</tr>
<tr>
<td>Printing</td>
<td>22</td>
<td>457</td>
<td>51%</td>
<td>37%</td>
</tr>
<tr>
<td>Chemical, Fibers</td>
<td>24</td>
<td>614</td>
<td>43%</td>
<td>34%</td>
</tr>
<tr>
<td>Rubber and Plastic</td>
<td>25</td>
<td>717</td>
<td>21%</td>
<td>0%</td>
</tr>
<tr>
<td>Non metallic products</td>
<td>26</td>
<td>823</td>
<td>37%</td>
<td>7%</td>
</tr>
<tr>
<td>Metals</td>
<td>27</td>
<td>614</td>
<td>35%</td>
<td>19%</td>
</tr>
<tr>
<td>Metallic products</td>
<td>28</td>
<td>1183</td>
<td>61%</td>
<td>15%</td>
</tr>
<tr>
<td>Mechanical Products</td>
<td>29</td>
<td>2031</td>
<td>22%</td>
<td>30%</td>
</tr>
<tr>
<td>Electrical Products</td>
<td>31</td>
<td>522</td>
<td>21%</td>
<td>10%</td>
</tr>
<tr>
<td>Television and comm.</td>
<td>32</td>
<td>303</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td>Precision instruments</td>
<td>33</td>
<td>191</td>
<td>26%</td>
<td>25%</td>
</tr>
<tr>
<td>Vehicles</td>
<td>34</td>
<td>261</td>
<td>24%</td>
<td>59%</td>
</tr>
<tr>
<td>other manufacturing</td>
<td>36</td>
<td>664</td>
<td>30%</td>
<td>8%</td>
</tr>
</tbody>
</table>

demonstrates that the selection criterion includes in the two groups firms from nearly every 2 digit sector.

Figure 6 shows the percentage of financially constrained firms conditional on age for the whole sample and for the Constrained and Unconstrained groups. In this as well as in the following figures I smooth the age profiles with 9 year moving averages. The figure shows that financing frictions decrease with age for all firms. Importantly, the reduction is larger for the Constrained group. That is, this group identifies firms that are much more constrained when young related to the Unconstrained group, while the difference between groups is much reduced among older firms. These dynamics are consistent with the model, where firms face financing frictions and bankruptcy risk only when young.

In figure 7 I show total factor productivity as a function of age. I first calculate ag-
aggregate factor shares at the four digit level. Then I use these factor shares to calculate TFP (total factor productivity, based on added value) for each firm-year observation, and I use this series to compute the age profile of TFP. The figure shows that TFP is slightly decreasing for the overall sample, and that such decline is much more pronounced for the firms in the "Constrained" group than for those in the "Unconstrained" group.

While figure 7 shows a clear difference in the age profile of productivity among the two groups, it does not prove any causality from financing frictions to productivity growth along the firms life-cycle. It may be that the Constrained group simply selects sectors where firms are more likely to become more productive with age, for a variety of technological reasons. To at least partially control these technological factors, in figure 8 we first normalize TFP for each firm-year observation, dividing it by for the average TFP of firms with same age and in the same two digit sector. Then we use this "normalized" TFP to calculate the age profile of the two groups. Therefore
Figure 8 shows productivity over age relative to productivity over age at the two digit level. It confirms the difference between Constrained and Unconstrained group shown before, and is remarkably consistent with the productivity dynamics predicted by the simulations with risky (embodied) innovations (see figure 2).

The model generates this age profile of productivity because firms innovate more in the Unconstrained industry. Figure 9 shows the fraction of firms that innovate in the empirical sample, and confirms that firms in the Unconstrained group both innovate more on average and have a probability of innovation that grows more with the age of the firm.

The model also predicts that this age profile is caused by the risk of innovation. We can indirectly test this result by looking at the different types of innovation implemented by firms. Figure 10 shows the fraction of firms performing R&D to develop and introduce new products, which can be considered as a relatively risky innovation activ-
Figure 8: TFP over age relative to TFP over age at the two digit level.

Figure 9: Fraction of firms performing R&D
ity since new products have an uncertain outcome. The figure confirms the prediction of the simulation with risky innovation that Unconstrained firms innovate more.

Conversely, figure 11 shows the fraction of firms performing R&D to either improve the current products or the current productive processes, which can be considered as a relatively safe type of innovation activity. Consistently with the predictions of the model, this type of innovation activity is more frequent among firms in the Constrained group.

Finally, a necessary condition in the model to generate the above results is that financing frictions act as barriers to entry and increase the profitability of firms. Figure 12 verifies this hypothesis by showing average net income over total assets, for the constrained and unconstrained groups, relative to average net income over total asset at the two digits level. It confirms that profitability (conditional to two digit sector and age) is relatively higher for the Constrained than for the Unconstrained group.
Figure 11: Fraction of firms performing R&D to improve the current products and/or the current productive processes

Figure 12: Net income over assets relative to net income over assets at the two digit level.
5.1 Conclusions

To be added

5.2 References

To be added

6 Computational appendix

To be added

7 Empirical Appendix

To be added