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OPTIMAL EXCHANGE RATE POLICY UNDER COLLATERAL CONSTRAINTS AND WAGE RIGIDITY

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Optimal Exchange Rate Policy Under Collateral Constraints and Wage Rigidity

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Abstract

Existing literature on optimal exchange rate policy in small open economies has studied wage rigidities and collateral constraints separately, yielding opposing policy recommendations. This paper shows that in a model that simultaneously includes both frictions, exchange rate policy faces a "credit access – unemployment trade-off," i.e. the choice between reducing involuntary unemployment and relaxing the external debt limit. We show that optimal exchange rate policy always responds to negative shocks by depreciating the nominal exchange rate. If the collateral constraint does not bind, optimal exchange rate depreciation achieves full-employment. If the collateral constraint does bind, optimal exchange rate depreciation is more contained than the depreciation that achieves full-employment and resulting unemployment is 11% on average. Fixed exchange rate regimes are inefficient in managing the debt-unemployment trade-off, and imply sizable welfare costs in all states with respect to the full-employment policy – 4 percent on average. Welfare costs of the full-employment policy are on average small (0.08 percent) but can achieve non-trivial levels (of 1 percent) in states with high debt and negative external conditions.

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1 Introduction

During external crises, exchange rate policy in emerging economies seems to imply choosing "between a rock and a hard place." Typical examples are the policy debates during the East Asian and Latin American crises of the late 90s (see Stiglitz, 2002, Calvo, 1999, Fischer, 1998) and the peripheral European debt crises that started in 2008 (see, for example, Krugman, 2010 and Feldstein, 2011). On the one hand, it is argued that fixing the exchange rate leads to unemployment and a slow and costly recessionary adjustment for the economy. On the other hand, it is claimed that, given that many of these economies have liabilities denominated in foreign currency, an exchange rate depreciation would produce sizable debt revaluation in terms of income, balance-sheet effects and financial destabilization.

We will call this the "credit access – unemployment trade-off" of exchange rate policy. The objective of this paper is to determine optimal exchange rate policy under this trade-off, by combining two recent branches of theoretical literature that have studied the two phenomena separately. The first branch is the literature on downward nominal wage rigidities initiated by Schmitt-Grohe and Uribe (2011, 2012). This literature has shown the significant costs, in terms of unemployment and welfare, of fixing the nominal exchange rate during external crises. In particular, the combination of two nominal rigidities – the factor price rigidity and the fixed exchange rate – creates a real wage rigidity that generates an inefficient and costly adjustment to negative shocks, with involuntary unemployment. The authors show that currency pegs can quantitatively account for the high increase in unemployment observed during episodes such as the Argentinean crisis of 2001 or the peripheral Europe crisis of 2008. It follows that the optimal exchange rate policy during external crisis periods is to depreciate nominal and real exchange rates in order to deflate the real value of wages and restore full employment in the economy.

The second branch of the literature, is that in which external borrowing is limited by the value of a collateral in the form of tradable and nontradable output as in Mendoza (2002) and Bianchi (2011). The literature that analyzes exchange rate policy under this collateral constraint has stressed the costs, in terms of deleveraging and consumption adjustment, associated with real exchange rate depreciations during external crisis periods. In particular, in an economy with debt denominated in the international unit of account, real exchange rate depreciations are associated with revaluations of external debt in terms of income and thus may imply a tightening in the collateral constraint and a large cost in terms of consumption adjustment. For instance, Mendoza (2005), concludes that policy-induced changes in relative prices can lead to a downward spiral, by the combination of balance-sheet effects and Fisherian debt-deflation. Similarly, Benigno et al. (2012) conclude in this framework that optimal real exchange rate policy consists of "leaning against the wind."

This paper combines these two branches of the literature and determines the optimal exchange rate policy in a standard two sector (tradable and nontradable) small open economy model that incorporates both frictions: the downward nominal wage rigidity and the collateral constraint in the form of tradable and nontradable output.

The contribution of this paper is threefold. First, we construct an environment that provides a theoretical justification for credit access – unemployment trade-off. In particular, we show that in this economy credit access and unemployment are two separate welfare objectives, and that there exists a tension between the two objectives, justifying the exchange rate policy debate that is typically observed during financial crises in emerging economies. The tension between the two policy objectives stems from the fact that for typical parameter values of the intratemporal elasticity of substitution between tradable and nontradable goods, the collateral constraint is decreasing in the level of employment. Hence, the policy maker might need to choose between a lower level of unemployment with a tighter collateral constraint and higher unemployment with a looser collateral constraint.

The second contribution of this paper is to show that, even in the presence of this trade-off, exchange rate depreciations are always optimal during external crises. Fixing the exchange rate is an inefficient way to deal with the credit constraint: resulting unemployment is much higher than the one that is necessary to maintain optimal credit access. As a consequence, the welfare costs of unemployment that result from fixing the nominal exchange rate significantly outweigh welfare gains from relaxing the collateral constraint during crises. The welfare gains of the full-employment policy with respect to the currency peg are sizable (3% of consumption per period on average). Moreover, we find that a "full-employment" exchange rate regime (in which exchange rate policy ensures no unemployment at all times) dominates, in terms of welfare, the fixed exchange rate regime for all states.

Finally, under binding credit constraint the optimal exchange rate depreciation is smaller than the one that would achieve full employment. As a result a sizable unemployment (11% on average) emerges under the optimal policy when the credit constraint binds. Welfare costs of the full-employment policy with respect to the optimal exchange rate policy are relatively small (0.06 percent of period consumption on average) but can reach a non-trivial size (1.03 percent of consumption per period) during states with high debt and negative external conditions. Optimal exchange rate policy is consistent with "managed-floating" exchange rate regime, so often observed in emerging economies during financial crises (see, for example, Klein and Shambaugh, 2009)

The presence of the credit access – unemployment trade-off is a key difference between this paper and previous studies on exchange rate policy that combines nominal and financial frictions. First, in a part of this literature credit access is not an objective in itself, but rather is relevant insofar as it affects employment and output. This is the case for instance of Cespedes et al. (2004) and Roldos et al (2007). Second, in part of previous literature, exchange rate depreciations do not affect or even improve the borrowing capacity in the economy, a case parallel to that of the "divine coincidence," where stabilizing inflation also stabilizes the output gap (Blanchard and Gali, 2005). For instance, in Cespedes et al. (2004), the financial friction takes the form of the financial amplificator (as in Bernanke et al., 1999). In this framework, a real depreciation is negative for the risk premium due to balance-sheet effects, but is also positive for the risk premium due to the increase in employment. The authors conclude that "the net result is that the behavior of the risk premium is independent of the exchange rate regime." In Fornaro (2012), the financial friction takes the form of a collateral constraint limiting the size of external debt to a fraction of the market value of an asset (e.g. land). In this setup, the net effect of exchange rate depreciations is to help stabilize the economy by preventing asset prices from falling too much. In the setup of the present paper, employment and borrowing conditions for the economy enter into a direct contradiction, and therefore the model provides a stronger alternative to flexible exchange rates.

We consider it relevant to determine the optimal exchange rate policy choice facing the credit access – unemployment trade-off for at least two reasons. First, the policy choice in this trade-off is both a long-standing and still lively policy debate. J. M. Keynes, for instance, was first actively opposed to the return of Britain to the gold standard after World War I, arguing that it would have a very high cost in terms of unemployment (see Keynes, 1925). However, when the Great Depression started, Keynes recommended against devaluation, claiming that now the costs in terms of debt revaluation and financial destabilization would outweigh the benefits (see Irwin, 2011). In the same line, Diaz-Alejandro (1966), analyzing Argentina's exchange rate policy in the 1950s, highlighted the possibility that devaluations lead to negative wealth effects and adjustment in consumption from balance-sheet effects. This policy debate, was triggered again by the crisis of peripheral Europe that started in 2008, in which there is, simultaneously, high unemployment and high debt levels denominated in euros.

Second, the empirical literature shows that both concerns are justified by evidence. Cross-country regressions for emerging economies tend to show both that fixing the exchange rate during financial crisis episodes is associated with larger output contractions (see, for example, Ortiz et al., 2009) and that balance-sheet effects play a key role in determining the access to international credit markets (see, for example, Calvo et al., 2009). This highlights the relevance of evaluating this policy trade-off with a structural model that allows the construction of counterfactuals and welfare comparisons of different exchange rate regimes.

The paper is organized as follows. Section 2 presents the model economy. Section 3 describes the exchange rate policy trade-off that emerges in this economy. Section 4 characterizes three possible exchange rate regimes in this setup: the optimal, the full-employment and the fixed exchange rate regimes. Section 5 presents the quantitative analysis comparing the aggregate dynamics and welfare under the three regimes. Finally, section 6 concludes.

2 The Model Economy

This section describes the model economy that will be used to conduct exchange rate policy analysis. We consider a dynamic, stochastic, small open economy model with a downward nominal wage rigidity, as the one developed in Schmitt-Grohe and Uribe (2011), and incorporate a collateral constraint as in Mendoza (2002), Bianchi (2010) and Benigno et al. (2009). The economy only has access to a one period, non-state contingent debt instrument, denominated in units of tradable goods, capturing *liability dollarization*[1]. The model then features a nominal rigidity and two financial frictions that will interact to determine the exchange rate policy trade-off.

The model has two types of goods: tradables and nontradables. Tradables are endowed to the economy, and their price is determined by the law of one price. Nontradables are produced by the small open economy, and their price is determined by domestic demand and supply. Fluctuations in the small open economy are driven by exogenous shocks to tradable endowment and to the interest rate on external debt.

[1] Liability dollarization or "original sin" is a phenomonenon that has been widely documented to lie at the heart of balance-sheet effects in emerging economies (see Eichengreen, Hausmann and Panizza, 2005).

2.1 Households

Households' preferences over consumption are described by the expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right) \tag{1}$$

where c_t^T denotes consumption, the function $U\left(.\right)$ is assumed to be strictly increasing and concave, $\beta \in (0,1)$, and $\mathbb{E}_t$ denotes expectation conditional on the information set available at time t.

The consumption good is assumed to be a composite of tradables and nontradable goods, with a CES aggregation technology:

$$c_t = A\left(c_t^T, c_t^N\right) \tag{2}$$

where c_t^T denotes tradable consumption, c_t^N denotes nontradable consumption and $A\left(c^T, c^N\right) = \left[a\left(c^T\right)^{1-\frac{1}{\xi}} + (1-a)\left(c^N\right)^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$.

Each period, households receive a stochastic endowment (y_t^T) and profits from the ownership of firms (Π_t). They supply inelastically $\overline{h}$ hours of work to the labor market. Due to the presence of the wage rigidity (discussed in detail in the next sections), households will only be able to sell $h_t \leq \overline{h}$ hours in the labor market. Actual hours worked, h_t, is determined by firms and is taken as given by the households.

Households have access to a one period, non state contingent bond denominated in units of tradable goods, that can be traded internationally paying an exogenous and stochastic gross interest rate R_t. The model therefore assumes full liability dollarization. Debt acquired in period t is taxed at rate τ_t^d. Households' sequential budget constraint is therefore given by:

$$\frac{d_{t+1.}}{R_t}\left(1-\tau_t^d\right) \geq d_t + c_t^T + p_t c_t^N - \left(y_t^T + w_t h_t + \Pi_t\right) - T_t \tag{3}$$

where d_{t+1} denotes the level of debt assumed in period t and due in period $t+1$, $p_t \equiv \frac{P_t^N}{P_t^T}$ denotes the relative price of nontradables in terms of tradables, w_t denotes the wage rate in terms of tradable goods and T_t denotes a lump sum transfer in period t.

Finally, households face a collateral constraint as in Mendoza (2002, 2005), Bianchi (2011) and Benigno et al. (2009) of the form[2]:

[2] Korinek (2011) shows that this form of the collateral contraint can be rationalized as a renegoziation-proof form of debt contract in an imperfect credit market in which households can renegotiate external debt and lenders can extract at most a fraction κ of the current income of borrowers if debt is renegociated.

$$d_{t+1.} \leq \kappa \left(y_t^T + w_t h_t + \Pi_t \right) \tag{4}$$

where $\kappa > 0$, $\kappa \left(y_t^T + w_t h_t + \Pi_t \right) \leq d^N$ and d^N is the natural debt limit.

The household problem is to choose state-contingent plans for c_t, c_t^T and c_t^N in order to maximize the expected utility (1) subject to the consumption aggregation technology (2), the sequential budget constraint (3), and the collateral constraint (4), for a given value of initial debt level d_0, for the given sequence of prices w_t and p_t, for the given sequence of hours worked h_t and profits Π_t^N, stochastic tradable endowment y_t^T and interest rate R_t, and for the given sequence of policies τ_t^d and T_t.

Denoting λ_t the Lagrange multiplier associated with the budget constraint (3) and μ_t the Lagrange multiplier associated with the collateral constraint (4), the optimality conditions are (2), (3) holding with equality, (4), the first order conditions:

$$\begin{aligned}
\lambda_t R_t^{-1} \left(1 - \tau_t^d \right) &= \beta \mathbb{E}_t \lambda_{t+1} + \mu_t && \text{(H.1)} \\
U_c A_T \left(c_t^T, c_t^N \right) &= \lambda_t && \text{(H.2)} \\
\frac{A_N \left(c_t^T, c_t^N \right)}{A_T \left(c_t^T, c_t^N \right)} &= p_t && \text{(H.3)}
\end{aligned}$$

and the complementary slackness conditions:

$$\mu_t \geq 0, \mu_t \left(\kappa \left(y_t^T + w_t h_t + \Pi_t^N \right) - d_{t+1.} \right) = 0 \tag{CS.1}$$

2.2 Firms

Each period, operating in competitive labor and product markets, firms hire labor to produce the nontradable good, y_t^N. Profits each period are given

$$\Pi_t = p_t F \left(h_t \right) - w_t h_t$$

where the production function $F(.)$ is assumed to be increasing and concave.

The firms' problem is to choose h_t to maximize profits given prices p_t and w_t. The first-order condition of this problem is :

$$p_t F' \left(h_t \right) = w_t \tag{F.1}$$

This condition implicitly defines the demand of labor from firms.

2.3 The Labor Market

Nominal wages (W_t) are assumed to be downwardly rigid as in Schmitt-Grohe and Uribe (2011, 2012)[3]:

$$W_t \geq \gamma W_{t-1}$$

for $\gamma > 0$.

Assuming the law of one price holds for tradable goods and that the foreign currency price of tradable goods (P_t^{T*}) is constant and normalized to one, wages in terms of tradable goods (w_t) can be expressed as:

$$w_t = \frac{W_t}{E_t}$$

where E_t is the nominal exchange rate.

From this we can express the wage rigidity as follows:

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t} \tag{LM.1}$$

where ϵ_t is the gross depreciation of the nominal exchange rate defined as $\epsilon_t \equiv \frac{E_t}{E_{t-1}}$.

Actual hours worked cannot exceed the inelastically supplied level of hours:

$$h_t \leq \overline{h} \tag{LM.2}$$

When the nominal wage rigidity binds, the labor market can exhibit involuntary unemployment, given by $\overline{h} - h_t$. This means the following slackness condition must hold at all dates and states:

$$\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) \left(\overline{h} - h_t \right) = 0 \tag{LM.3}$$

This condition means that when the nominal wage rigidity is not binding, the labor market must exhibit full employment, and if it exhibits unemployment, it must be the case that the nominal wage rigidity is binding.

[3] Schmitt-Grohe and Uribe (2011) show that this form of wage rigidity is supported both by macro evidence during external crises in emerging economies, and by micro evidence for the US (see, for example Barattieri, Basu and Gottschalk (2010)).

2.4 The Government

The government imposes a proportional tax (subsidy) on debt τ_t^d that is rebated lump sum to households (T_t), to balance its budget each period:

$$\frac{d_{t+1.}}{R_t}\tau_t^d = T_t \tag{G.1}$$

In the next section we will discuss how this tax is determined.

2.5 General Equilibrium Dynamics

The market for nontradable goods clears at all times, from which

$$c_t^N = F\left(h_t\right) \tag{MC}$$

Combining condition (MC) with households' budget constraint (3) holding with equality, the definition of firms' profits, and the government's budget constraint (G.1) we obtain the resource constraint:

$$\frac{d_{t+1.}}{R_t} = d_t + c_t^T - y_t^T \tag{RC}$$

Using firms' profit definition we can re-express the collateral constraint (4) as follows:

$$d_{t+1.} \leq \kappa\left(y_t^T + p_t F\left(h_t\right)\right) \equiv \overline{d}_t \tag{CC}$$

The general equilibrium dynamics is then given by stochastic processes $c_t^T, h_t, w_t, d_{t+1}, \lambda_t, \mu_t$ satisfying the set of equations (GE):

(RC): $d_{t+1.}R_t^{-1} = d_t + c_t^T - y_t^T$
(H.1): $\lambda_t R_t^{-1}\left(1-\tau_t^d\right) = \beta\mathbb{E}_t\lambda_{t+1} + \mu_t$
(H.2): $U_c\left(c_t^T, F\left(h_t\right)\right) A_T\left(c_t^T, F\left(h_t\right)\right) = \lambda_t$
(CC): $d_{t+1.} \leq \kappa\left(y_t^T + \frac{A_N\left(c_t^T,F(h_t)\right)}{A_T\left(c_t^T,F(h_t)\right)}F\left(h_t\right)\right)$
(CS.1): $\mu_t \geq 0, \mu_t\left(\kappa\left(y_t^T + \frac{A_N\left(c_t^T,F(h_t)\right)}{A_T\left(c_t^T,F(h_t)\right)}F\left(h_t\right)\right) - d_{t+1.}\right) = 0$
(LM.1): $w_t \geq \gamma\frac{w_{t-1}}{\epsilon_t}$
(LM.2): $h_t \leq \overline{h}$
(LM.3): $\left(w_t - \gamma\frac{w_{t-1}}{\epsilon_t}\right)\left(\overline{h} - h_t\right) = 0$
(F.1): $\frac{A_N\left(c_t^T,F(h_t)\right)}{A_T\left(c_t^T,F(h_t)\right)}F'\left(h_t\right) = w_t$
(G.1): $T_t = \tau_t^d d_{t+1}R_t^{-1}$

given the exogenous processes y_t^T and R_t, an exchange rate policy ϵ_t, a capital control policy τ_t^d and initial conditions w_{-1} and d_0.

Note that the only differences between this model economy and the one in Schmitt-Grohe and Uribe (2011, 2012) are equations (CC) and (CS.1). In their case, the debt limit is given by the natural debt limit instead of the collateral constraint.

3 The Credit Access – Unemployment Trade-Off

This section describes the trade-off that exchange rate policy faces in an economy with a collateral constraint, in the form of tradable and nontradable output, and a downward nominal wage rigidity. Parallel to the traditional *inflation-unemployment* trade-off in the New Keynesian literature, the exchange rate policy in this economy may face what we can call a "credit access – unemployment trade-off:" under a binding collateral constraint and wage rigidity, changes in exchange rate policy push employment and the external credit constraint in opposite directions.

Under a utility based criterion for exchange rate policy (Woodford, 2003), two things need to be established about this policy trade-off: first, why credit access and unemployment are two independent objectives, and second, why exchange rate policy implies a tension between the two objectives.

Credit Access and Unemployment as Welfare Objectives

In most of the previous literature on exchange rate policy that integrates nominal rigidities and financial frictions, credit market access is not an objective in itself, but rather is relevant insofar as it affects employment and output. This is the case, for instance, in Cespedes et al. (2004), which studies an economy featuring the financial amplificator (Bernanke et al., 1999), or in Roldos et al. (2007), which analyzes an economy with credit-constrained firms. In this literature, the central stabilization policy objective is employment and output, and the discussion is on whether exchange rate depreciations are in fact expansionary or contractionary.

The present paper describes an environment in which credit market access is an independent objective of employment. Under a binding credit constraint, the tighter the constraint for a given debt level, the higher the debt repayment and the adjustment required of the household's consumption in order to repay debt. Thus, credit market access directly affects tradable consumption and welfare (as in Mendoza, 2005, Bianchi 2011 and

Beningo et al. 2012). As usual, welfare is also affected by unemployment, making production and consumption are inefficiently low. Thus, a welfare criterion for exchange rate policy includes two stabilization objectives: credit access and employment. We establish this formally in the following proposition.

Proposition 1 (Welfare criterion). *In an equilibrium at time t in which the collateral constraint (CC) is strictly binding ($\mu_t > 0$), there exist unemployment ($h_t < h$) and $\tau_t^d \geq 0$, the value function of the representative households' problem can be represented as:*

$$V_t = U(A(\underbrace{\overline{d_t}}_{\textit{credit-access objective}} R_t^{-1} - d_t + y_t^T, \quad \underbrace{F(h_t)}_{\textit{employment objective}} \quad)) + \beta \mathbb{E}_t V_{t+1}$$

with $\left(\frac{\partial V_t}{\partial \overline{d_t}}\right)_{h_t} > 0$ *and* $\left(\frac{\partial V_t}{\partial h_t}\right)_{\overline{d_t}} > 0$

Proof. See Appendix A ■

This proposition shows that, under a binding credit constraint, the value function of the households can be represented as a function only of the credit limit and employment. If the policy maker could increase employment *without* affecting the collateral constraint or relax the collateral constraint *without* affecting employment, this would improve the households' welfare. The next section explores the relationship between the two stabilization objectives.

The Trade-Off between Credit Access and Unemployment

We now focus on why exchange rate policy might imply a tension between the two stabilization objectives. The mechanism is as follows. Under binding nominal downward wage rigidity, a depreciation of the nominal exchange decreases real wages and, thus, helps achieve the objective unemployment reduction. But it is also associated with a *real* exchange rate depreciation, which decreases the value of non-tradable output in units of tradables. Recall that the collateral in this economy is given by the value, in units of tradables, of tradable and non-tradable output. Thus, if the price effect (real exchange rate depreciation) dominates the quantity effect (employment increase), an exchange rate depreciation can decrease the collateral value and make the credit limit tighter, which goes against the credit access objective.

To show this formally we begin by making the following two parametric assumptions:

Assumption 1 The intratemporal elasticity of substitution between tradable and non-tradable goods in less than one: $\xi < 1$

Assumption 2 In any equilibrium, $\kappa p_{c_t^T}(c_t^T, h_t) R_t^{-1} < 1$

Assumption 1 is the key parametric assumption necessary for the existence of a credit access – unemployment trade-off. If it is satisfied, the price effect of an exchange rate depreciation dominates the quantity effect resulting in a tighter collateral constraint. At the end of this section we show that empirical evidence for emerging economies provides wide support for Assumption 1. Assumption 2 is a technical assumption on the parameter κ, which governs the capacity of nontradable output to be used as collateral, used in several studies featuring a collateral constraint in the form of tradable and nontradable income (see for example Jeanne and Korinek, 2011). This assumption ensures that if the collateral constraint is binding for a certain debt level, it is not feasible for the economy to increase debt. Under these two assumptions we can establish the following proposition.

Proposition 2 (Trade-off)**.** *Under Assumptions 1 and 2, around an equilibrium at time t in which the collateral constraint (CC) is strictly binding ($\mu_t > 0$) and there exist unemployment ($h_t < h$), there exist a negative relationship between the credit limit and employment, i.e.* $\frac{\partial \bar{d}_t}{\partial h_t} < 0$

Proof. See Appendix A ■

From this proposition, it follows that if the intratemporal elasticity of substitution is *less* than one ($\xi < 1$), depreciating the nominal exchange rate under binding collateral constraints will simultaneously increase employment and make the credit constraint tighter. Therefore, this provides a theoretical justification for the existence of the exchange rate policy debate that is typically observed during financial crises in emerging economies, which weighs the two policy objectives. The policy maker in this economy might need to choose between a lower unemployment rate, a tighter collateral constraint and a lower debt level on the one hand, and a higher unemployment rate, a looser collateral constraint and higher debt level on the other hand.

From Proposition 2, it also follows that if the intratemporal elasticity of substitution is greater than or equal to one ($\xi \geq 1$), the credit access – unemployment trade-off vanishes. In particular, if it is *equal* to one ($\xi = 1$), employment does not influence the collateral constraint. This is the result obtained by Cespedes et al. (2004), where, in the framework of the financial amplificator (Bernanke et al., 1999), the behavior of the risk premium ends up being independent of the exchange rate regime. Moreover, if the intratemporal elasticity of substitution is *greater* than one ($\xi > 1$), the credit-unemployment trade-off overturns and a decrease in unemployment also helps relax the collateral constraint. This would be a case parallel to that of the "divine coincidence," where stabilizing inflation also stabilizes the output gap (Blanchard and Gali, 2005). Fornaro (2012) obtains this result using a financial friction in the form of a collateral constraint limiting the size of external debt to a fraction of the market value of an asset (e.g. land). Exchange rate depreciations increase employment and help stabilize the economy by preventing asset prices from falling too much.

Therefore, the presence of the credit access – unemployment trade-off is a major difference between the present paper and previous literature and provides a stronger alternative to flexible exchange rates.

Empirical Evidence on the Intratemporal Elasticity of Substitution

As shown in Proposition 2, the key parameter that determines the existence of a credit access – unemployment trade-off is the intratemporal elasticity of substitution between tradable and non-tradable goods. In particular, there exists a tension between credit access and unemployment only if the elasticity of substitution is less than one (if Assumption 1 is satisfied). If this is the case, tradable and non-tradable goods are gross complements, and the price effect (real exchange rate depreciation) associated with increasing employment dominates the quantity effect. As a result, an exchange rate depreciation can decrease the collateral value and make the credit limit tighter.

There is a wide support from the empirical literature for Assumption 1. In a sample of developed and emerging economies, Stockman and Tesar (1995), estimate a value of the elasticity of substitution between tradable and nontradable goods of 0.44. Separating the sample in developed and emerging economies, Mendoza (1995) finds values of the elasticity of 0.74 and 0.43 respectively. In studies for emerging economies, Gonzalez-Rozada et al. (2004) found estimates in the range of 0.4 and 0.48 for Argentina and Lorenzo et al (2005), found estimates in the range of 0.46 and 0.75 for Uruguay. [4]

Moreover, following this empirical literature, all the studies referenced in the present paper that calibrate a two sector small open economy, use a parameter value of the elasticity of substitution in the range between 0.44 and 0.83.

[4] Ostry and Reinhart (1992) found evidence inconclusive in this respect with estimates between 0.66 and 1.44 depending on the emerging region and the instrumental variable considered. For a survey on the methodologies used to estimate the elasticity of substitution between tradable and nontradable goods see Akinci (2011).

4 Exchange Rate Regimes

We define three possible exchange rate regimes: the optimal, the full-employment, and the currency peg. We evaluate exchange rate regimes, conditional on an optimal capital control policy. As shown in Bianchi (2011) and Benigno et al. (2009), the collateral constraint used in this paper may induce *overborrowing* or *underborrowing*.[5] We use the tax (subsidy) on debt τ_t^d to correct the inefficiency stemming from the collateral constraint. This is typically done in the New Keynesian literature, where imposing a tax (subsidy) eliminates the distortion stemming from the monopolistic competition (see, for example, Woodford, 2003). Evaluating different exchange rate policies without using this capital control is part of ongoing research.

4.1 Optimal Exchange Rate Regime

We begin considering the optimal exchange rate policy with optimal capital control tax, defined as follows:

Optimal exchange rate policy with optimal capital control tax (OP) : Processes $\left\{\epsilon_t, \tau_t^d\right\}$ that maximize household's expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (RC),(H.1-2), (CC), (CS.1), (LM.1-3), (F.1), (G.1).

To characterize the allocation under optimal exchange rate and capital control policy, we set up the Ramsey problem dropping constraints (H.1-2), (CS.1), (LM.1-3), (F.1), (G.1). In Appendix B we show that the solution satisfies the omitted constraints. The Ramsey problem is then to maximize (1) with respect to $\left\{d_{t+1}, c_t^T, h_t\right\}$, subject to (RC), (CC) and (LM.2). The Lagrangian of the Ramsey problem is then given by:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} U\left(A\left(c_t^T, F\left(h_t\right)\right)\right) + \\ +\phi_t^F \left[\frac{d_{t+1}}{R_t} - d_t + y_t^T - c_t^T\right] + \\ +\phi_t^{\mu} \left[\kappa \left(y_t^T + \left(\frac{1-a}{a}\right) \left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{1-\frac{1}{\xi}}\right) - d_{t+1}\right] + \\ +\phi_t^W \left[\overline{h} - h_t\right] \end{array} \right\}$$

[5] Overborrowing (underborrowing) refers to a situation in which the social planner optimal average level of external debt is higher (lower) than the one of the descentralized equilibrium.

The optimality conditions associated with this problem are (RC), (CC), (LM.2), the first order conditions:

$$\frac{\phi_t^F}{R_t} = \beta \mathbb{E}_t \phi_{t+1}^F + \phi_t^\mu \tag{FOC.1}$$

$$\phi_t^F = U_{c,t} A_{T,t} + \phi_t^\mu \kappa \left(\frac{1}{\xi}\right) \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{F\left(h_t\right)}\right)^{\frac{1}{\xi}-1} \tag{FOC.2}$$

$$\phi_t^W = F_{h,t} \left[U_{c,t} A_{N,t} + \phi_t^\mu \left(\frac{\xi - 1}{\xi}\right) \kappa \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{F\left(h_t\right)}\right)^{\frac{1}{\xi}} \right] \tag{FOC.3}$$

and the complementary slackness conditions:

$$\begin{aligned} \phi_t^\mu &\geq 0; \phi_t^\mu \left[\kappa \left(y_t^T + \left(\frac{1-a}{a}\right) \left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{1-\frac{1}{\xi}} \right) - d_{t+1} \right] = 0 \\ \phi_t^W &\geq 0; \phi_t^W \left[\overline{h} - h_t\right] = 0 \end{aligned} \tag{CS.2-3}$$

From the optimality conditions of the Ramsey problem we can show the following proposition:

Proposition 3 (No unemployment under no binding collateral constraint)**.** *In any period in which the collateral constraint does not bind ($d_{t+1} < \overline{d}_t$), optimal exchange rate policy features full-employment ($h_t = \overline{h}$).*

Proof. See Appendix A ■

This proposition characterizes the optimal exchange rate policy in all periods in which the collateral constraint is not binding. If the economy experiences a negative shock that decreases the full-employment real wage, but the collateral constraint is not binding, optimal policy implies depreciating the exchange rate to liquate real wages and maintain full-employment. An intuition behind this result is that, given that the social planner has a capital control tax, exchange rate policy does not need to have any precautionary motive.

The optimal exchange rate policy can be characterized by the following Bellman equation:

$$V^{OP}\left(y_t^T, R_t, d_t\right) = \max_{d_{t+1}, c_t^T, h_t} \left[U\left(A\left(c_t^T, F\left(h_t\right)\right)\right) + \beta \mathbb{E}_t V^{OP}\left(y_{t+1}^T, R_{t+1}, d_{t+1}\right)\right] \quad \text{(BE.OP)}$$

$$\begin{aligned} s.t. \frac{d_{t+1}}{R_t} &= d_t + c_t^T - y_t^T && \text{(RC)} \\ d_{t+1} &\leq \kappa \left(y_t^T + \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{\frac{\xi-1}{\xi}}\right) && \text{(CC)} \\ h_t &\leq \overline{h} && \text{(LM.2)} \end{aligned}$$

where $V^{OP}\left(y_t^T, R_t, d_t\right)$ denotes the value function of households under optimal exchange rate and capital control policies. This formulation will be used in the quantitative analysis.

4.2 Full-Employment Exchange Rate Regime

In the second place, we consider an exchange rate policy aimed at maintaining full-employment at all states and dates that we refer to as the "full-employment policy" (as in Schmitt-Grohe-Uribe, 2011). Under the full-employment policy:

$$h_t = \overline{h} \quad \text{(FE)}$$

for every t. The full employment policy with optimal capital control tax can be defined as follows:

Full-employment exchange rate policy with optimal capital control tax (FE) : Processes $\left\{\epsilon_t, \tau_t^d\right\}$ that maximize household's expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (RC),(H.1-2), (CC), (CS.1), (LM.1-2), (F.1), (G.1), and the full employment constraint (FE).

To characterize the optimal allocation under the full-employment policy, we follow the same strategy as in the optimal exchange rate policy and drop constraints (H.1-2), (CS.1), (LM.1), (LM.3), (F.1), (G.1). In Appendix B, we show that the solution satisfies the omitted constraints. This allows us to express the dynamics under the full-employment exchange rate policy with optimal capital control tax with the following Bellman equation:

$$V^{FE}\left(y_t^T, R_t, d_t\right) = \max_{d_{t+1}, c_t^T} \left[U\left(A\left(c_t^T, F\left(\overline{h}\right)\right)\right) + \beta \mathbb{E}_t V^{FE}\left(y_{t+1}^T, R_{t+1}, d_{t+1}\right)\right] \quad \text{(BE.FE)}$$

$$s.t. \frac{d_{t+1}}{R_t} = d_t + c_t^T - y_t^T \quad \text{(RC)}$$

$$d_{t+1} \leq \kappa\left(y_t^T + \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(\overline{h}\right)^{\frac{\xi-1}{\xi}}\right) \quad \text{(CC)}$$

where $V^{FE}\left(y_t^T, R_t, d_t\right)$ denotes the value function of households under the full employment exchange rate policy with optimal capital control tax.

This formulation will be used in the quantitative analysis in the next section. Note that the dynamics of the economy under the full-employment policy is similar to the one in Bianchi (2011) but with a fixed nontradable consumption and a stochastic interest rate.

4.3 Fixed Exchange Rate Regime

Finally, we consider a policy aimed at keeping fixed the exchange rate at all states and dates, that we refer to as the fixed exchange rate policy or currency peg. Under a currency peg:

$$E_t = \overline{E} \quad \text{(CP)}$$

for every t. The fixed exchange rate policy with optimal capital control tax can be defined as follows:

Fixed exchange rate policy with optimal capital control tax (CP) : Processes $\left\{\epsilon_t, \tau_t^d\right\}$ that maximize household's expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (RC),(H.1-2), (CC), (CS.1), (LM.1-2), (F.1), (G.1), and currency peg constraint (CP).

To characterize the allocation under the currency peg with optimal capital control tax, we follow the same strategy as in the optimal exchange rate policy and drop constraints (H.1-2), (CS.1), (LM.3), (G.1). In Appendix B we show that the solution satisfies the omitted constraints. This allows us to express the dynamics under the currency peg with optimal capital control policy with the following Bellman equation:

$$V^{PEG}\left(y_t^T, R_t, d_t, w_{t-1}\right) = \max_{d_{t+1}, c_t^T} \left[U\left(A\left(c_t^T, F\left(\overline{h}\right)\right)\right) + \beta \mathbb{E}_t V^{PEG}\left(y_{t+1}^T, R_{t+1}, d_{t+1}, w_t\right)\right] \tag{BE.CP}$$

$$s.t. \frac{d_{t+1}}{R_t} = d_t + c_t^T - y_t^T \tag{RC}$$

$$d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{\frac{\xi-1}{\xi}}\right) \tag{CC}$$

$$w_t \geq \gamma w_{t-1} \tag{LM.1}$$

$$h_t \leq \overline{h} \tag{LM.2}$$

$$w_t = \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{-\frac{1}{\xi}} F'\left(h_t\right) \tag{F.1}$$

where $V^{PEG}\left(y_t^T, R_t, d_t, w_{t-1}\right)$ denotes the value function of households under the currency peg and optimal capital control tax.

5 Quantitative Analysis

The objective of this section is to quantitatively characterize the aggregate dynamics of the model economy under different exchange rate regimes. In particular, we want to determine the optimal exchange rate policy and compare its performance to that of the full-employment and fixed exchange rate regimes, both during periods of external crisis and under regular business cycle fluctuations. We begin by calibrating the model to Argentinean data and describing the computation method to solve the model numerically. We then compare the resulting ergodic distributions for external debt and unemployment under the three exchange rate regimes. Third, we compare the response of the economy during periods of external crisis. Finally, we compute welfare under the different exchange rate regimes.

5.1 Calibration and Computation

To characterize the aggregate dynamics under different exchange rate regimes, we solve numerically a calibrated version of the functional problems defined in Section 3. Due to the presence of occasionally binding constraints we resort to the method of value function iteration over a discretized state space.

As mentioned in Section 2, the consumption aggregator is assumed to be a CES aggregator. We further assume a CRRA period utility function, an isoelastic form for the production function:

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

$$F(h) = h^{\alpha}$$

We calibrate the model at the quarterly frequency, to match Argentinean data for the period 1983-2001. All the parameter values are shown in Table 1, indicating their source. We follow Schmitt-Grohe and Uribe (2011) in most parameter values of the calibration $\{\gamma, \sigma, a, \xi, \alpha, R\}$. Two parameters deserve special attention: the subjective discount factor β and the parameter κ governing the collateral constraint. The parameter β is typically used in the literature to match the observed average external debt-to-output ratio. Since all exchange rate policies are defined conditional on an optimal capital control tax, not observed in Argentina during the calibration period, instead of matching the historical average of external debt in the data (29.2 percent, Lane and Milesi-Ferreti, 2007), we use the findings in Bianchi (2011) and match the average external debt that results from optimal capital control tax (28.6 percent). The average of external debt is also influenced by the exchange rate regime. During our calibration period, Argentina alternated between different exchange rate regimes (see for example Reinhart and Rogoff, 2004). We match the average of external debt observed in the data with those generated by the model under the optimal exchange rate policy. Finally, we set the parameter κ to match the maximum debt-to-output ratio observed in the data (64.6 percent, Lane and Milesi-Ferreti, 2007) [6]

Table 1: Calibration

[6] Previous literature has used the parameter κ to match the frequency of *sudden stops*, defined as periods in which the collateral constraint is binding (see Bianchi (2011), Benigno et al (2009)). An advantage of targeting the maximum of debt-to-income ratio in this setup is that its value is independent of the exchange rate regime. Morevover, as shown in next section, the ocurrence of Sudden Stops resulting from this calibration is similar to the one obtained in Bianchi (2011).

Parameter	**Value**	**Description**
γ	0.99	Degree of downward nominal rigidity (SGU, 2011)
σ	5	Inverse of intertemporal elasticity of substitution (OR, 1992)
y_t^T	1	Steady-state tradable output
$\overline{h}$	1	Labor endowment
a	0.26	Share of tradables (SGU, 2011)
ξ	0.44	Intratemporal elasticity of substitution (GR, 2004)
α	0.75	Labor share of the nontradable sector (Uribe, 1997)
β	0.94725	Quarterly subjective discount factor (Calibrated)
R	0.0316	Steady-state interest rate (SGU, 2011)
κ	2.9	Limit of debt to income ratio (Calibrated)

Note. SGU: Schmitt-Grohe and Uribe; OR: Ostry and Reinhart; GR: Gonzalez Rosada et al.

We use the stochastic process for driving forces in the economy obtained in Schmitt-Grohe and Uribe (2011). The authors begin by assuming that tradable output and the interest rate follow a bivariate first-order autoregressive process of the form:

$$\begin{bmatrix} \ln\left(y_t^T\right) \\ \ln\left(\frac{R_t}{R}\right) \end{bmatrix} = A \begin{bmatrix} \ln\left(y_{t-1}^T\right) \\ \ln\left(\frac{R_{t-1}}{R}\right) \end{bmatrix} + \varepsilon_t$$

Second, using Argentinean data from 1983.I to 2001.IV they obtain OLS estimates of these matrices given by:

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \Sigma_\varepsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}$$

Finally, using this estimation, they discretize the AR(1) process using 21 equally spaced points for $\ln\left(y_t^T\right)$ and 11 equally spaced points for $\ln\left(\frac{R_t}{R}\right)$ and simulate the stochastic process for 2 million quarters to estimate the transition probability matrix.

To approximate the aggregate dynamics of the economy under the optimal and the full-employment policies, we discretize the endogenous state space (d_t) using 1001 equally spaced points. To approximate the dynamics under a currency peg, we use 251 equally spaced points for debt (d_t) and 250 equally spaced points for the log of previous period wage (w_{t-1}). Finally, we solve the functional equation (BE.OP) subject to (RC), (CC), (LM.2) to approximate the dynamics under the optimal exchange rate policy; the functional equation (BE.OP) subject to (RC), (CC) to approximate the dynamics under the full-employment policy; and the functional form (BE.CP) subject to (RC), (CC), (LM.1)-(LM.2) and (F.1) to approximate the dynamics under the currency peg . In the next sections we present the results for the quantitative analysis.

5.2 Distribution of External Debt and Unemployment

In this section we analyze how the choice of exchange rate regime affects the distribution of external debt and unemployment. Figure 2 depicts the unconditional distribution of external debt and unemployment under each of the exchange rate regimes. Table 2 presents the average, standard deviation and maximum values of each distribution.

Figure 2: Ergodic Distribution of External Debt and Unemployment

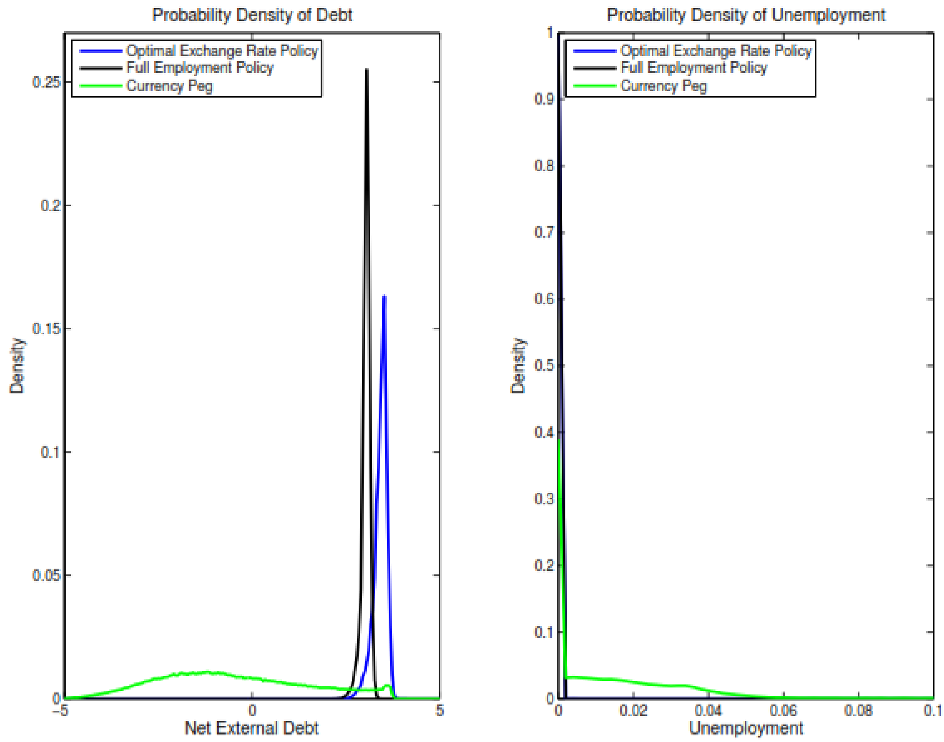


Table 2: Moments of External Debt and Unemployment

	Mean			St.D.			Max		
	OP	FE	CP	OP	FE	CP	OP	FE	CP
Unemployment rate (in %)	0.06	0	2.5	1.3	0	5.2	66.2	0	62.5
External Debt	3.5	3.1	−0.5	0.2	0.1	2.0	4.6	3.5	3.9
Debt-to-output ratio (in %)	28.5	24.6	−0.5	10.0	8.1	14.1	64.6	64.6	63.4

Two features characterize the optimal exchange rate policy. First, it entails a higher average and maximum level of external debt than the other two exchange rate regimes. To understand the connection between exchange rate regime and external debt distribution, we begin by showing how in an economy with a collateral constraint, the choice of exchange rate regime affects the *maximum* level of debt supported by the economy.

Let y_L^T be the lowest possible realization of tradable endowment and R_H the highest possible level of the interest rate. Then in any state in which there is a non-zero probability of visiting the state $\left(y_L^T, R_H\right)$, the maximum level of indebtness, d^C, such that with probability one, the economy respects the collateral constraint and nonnegative levels of consumption, can be formally defined as:

$$d^C = \left\{\max d \leq d^N : \mathcal{F}\left(d, h\right) \neq \varnothing\right\}$$

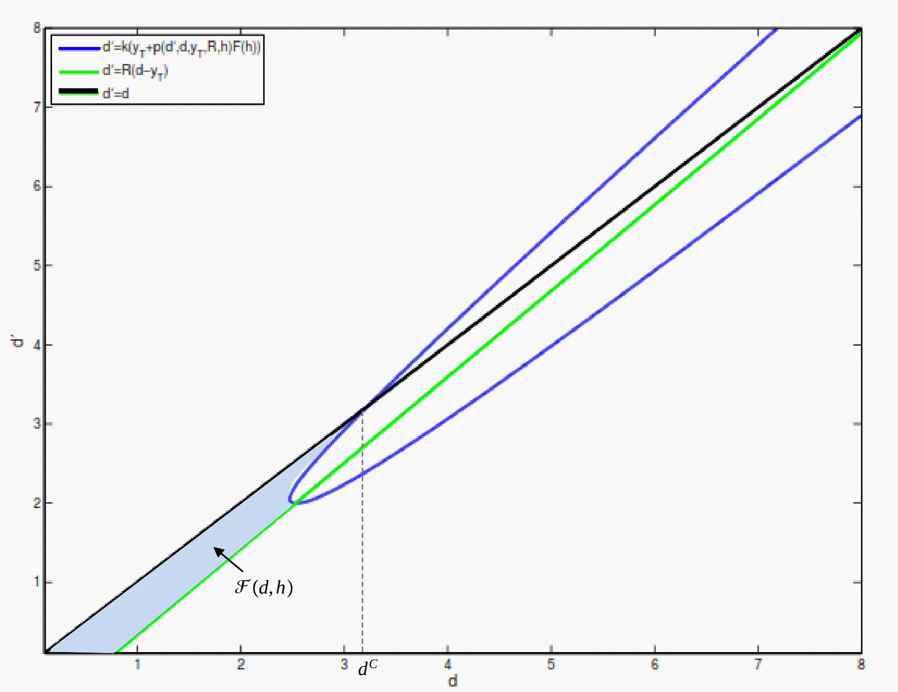
where d^N denotes the natural debt limit, and the set of feasible debt choices $\mathcal{F}\left(d, h\right)$ is defined by:

$$\mathcal{F}\left(d, h\right) = \left\{ \begin{array}{c} d' \leq d \ : d' \leq \kappa\left(y_L^T + \left(\frac{1-a}{a}\right)\left(\frac{d'}{R_H} + y_L^T - d\right)^{\frac{1}{\xi}}\left(F\left(h\right)\right)^{1-\frac{1}{\xi}}\right), \\ d' \geq R_H\left(y_L^T - d\right) \end{array} \right\}$$

Consider first this maximum level of external debt $\left(d^C\right)$ under the full-employment exchange rate regime. Figure 3 portrays the set $\mathcal{F}\left(d, \overline{h}\right)$ for our choice of parameter values (shaded region). The blue line indicates the collateral constraint, the green line zero consumption and the black line the 45 degree line. It can be seen that for current debt levels greater than 3.2, the set $\mathcal{F}\left(d, \overline{h}\right)$ is empty. This point defines the maximum level of external debt d^C, which is substantially lower than the natural debt limit (in our calibrated economy $d^N = 8.4$). Figure 2 and Table 2 show that the maximum level of external debt under full-employment is 3.5, very close to d^C[7].

[7] The maximum level in the distribution of external debt exceeds d^C due to the fact that in some states there is a zero probability of visiting the state $\left(y_L^T, R_H\right)$.

Figure 3: Maximum Level of External Debt under Full-Employment



Now we turn to the case of the optimal policy exchange rate policy, in which employment is a control variable. In this case, given Remark 1, the social planner can potentially choose any level of debt arbitrarily close to the natural debt limit. This is established in the following proposition:

Proposition 4. *If $\xi < 1$, for any $d < d^N$ there always exist an $h > 0$ such that the set $\mathcal{F}(d, h)$ is nonempty.*

Proof. See Appendix A ■

However, maintaining a level of debt substantially higher than the one of the full-employment exchange rate regime (close to the natural debt limit for example) would entail very high costs in terms of unemployment and nontradable consumption. As a consequence, the difference in the distribution of external debt for the two regimes is not that large. Figure 2 and Table 2 indicate that, the optimal exchange rate policy features an average and maximum level 13 percent and 31 percent higher than under the full-employment regime, respectively. As a reference, *without* a collateral constraint, our calibrated economy would display an average external debt 69 percent higher than under the full-employment policy with a collateral constraint and a maximum level of external debt of more than twice that of the full-employment policy with a collateral constraint, very close to the natural debt limit.

The second salient characteristic of optimal exchange rate policy is the other side of the coin: optimal exchange rate policy features practically no unemployment (0.06 percent on average). Although the maximum level of unemployment under the optimal exchange rate policy is remarkably high, this event is highly unlikely: the probability of experiencing an unemployment rate higher than 1 percent is 0.3 percent (See Figure 2). The low probability of high unemployment is a consequence of the fact that, with the optimal capital control tax, the frequency of *sudden stops* (defined as periods in which the collateral constraint is binding) is only 0.4 percent [8]. This extends the argument in Bianchi (2011) on the relevance of prudential policies in the presence of collateral constraints in the form of tradable and non-tradable output: under the optimal exchange rate policy, decreasing the probability of sudden stops decreases the probability of experiencing unemployment.

In the next section we analyze how high is unemployment during episodes of external crises (in which the probability of triggering binding collateral constraints is substantially higher).

Finally, the currency peg entails both lower levels of external debt and higher levels of unemployment than the full-employment and optimal exchange rate regimes. The reason why, under optimal capital control tax, the currency peg entails such low average levels of external debt is the one shown in Schmitt-Grohe and Uribe (2012): the downward nominal wage rigidity entails a pecuniary externality that creates a strong motive for optimal capital controls to be prudential in nature. Thus, the currency peg is an inefficient way to manage the debt-unemployment trade-off: for the same level of debt, unemployment is higher while, for the same level of unemployment, debt needs to be lower. In the last section we quantify the costs of this policy inefficiency in terms of welfare.

5.3 Dynamics during External Crises and Sudden Stops

In this section we characterize the dynamics of the economy under each exchange rate regime in response to large negative external shocks. Following Schmitt-Grohe and Uribe (2011), we define an *external crisis* as an episode in which tradable endowment is at or above trend in period t and at least two standard deviations below trend in period $t+10$.

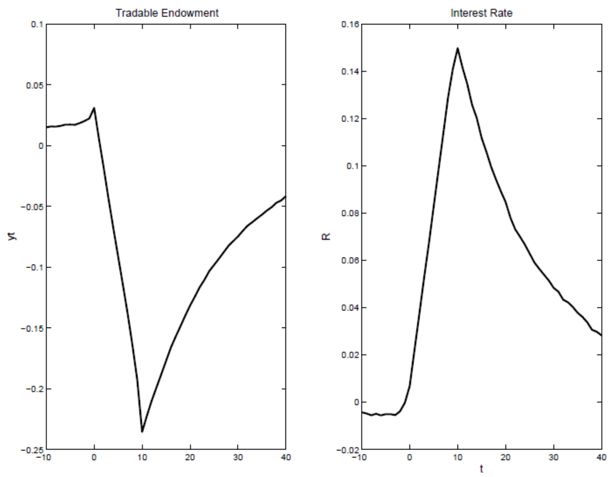
[8] The occurrence of Sudden Stop under our calibrated model is very similar to that of Bianchi (2011) under optimal capital control tax.

This definition is aimed at capturing extraordinary contractions such as the Argentinean 1989 and 2001 crises or the 2008 crisis in peripheral Europe.

To characterize the dynamics of the economy under the different exchange rate regimes, we follow a procedure similar to the one in Schmitt-Grohe and Uribe (2011). We simulate the model for 5 million quarters and identify episodes in which the behavior of external variables conforms to the definition of an external crisis. We then average the response of the variables, subtracting their sample means. We set $t = 0$ as the beginning of the crisis.

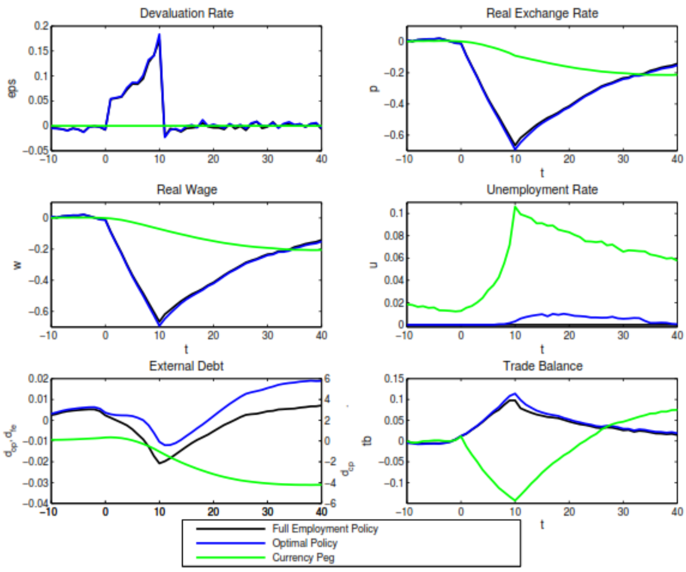
Figure 4 depicts the average external shock in a crisis episode. At the crisis trough ($t = 10$), tradable output is 24 percent below its trend, and the annual interest rate is 15 percentage point above its steady state value.

Figure 4: External Crises: Exogenous Variables



The average response of the nominal exchange rate and endogenous variables under the different exchange rate regimes is shown in Figure 5. Optimal and full-employment exchange rate policies have striking similarities, and display a sharp contrast with the response under a currency peg. Both optimal and full-employment policies imply a significant depreciation of the nominal exchange rate during external crises: 155 and 145 percent, from output peak to trough, respectively (45 and 43 percent annual rate). In the average crisis episode under the optimal exchange rate policy, the unemployment rate reaches a level of 1.1 percent, very low when compared to the 11 percent reached under the currency peg.

Figure 5: External Crises: Endogenous Variables



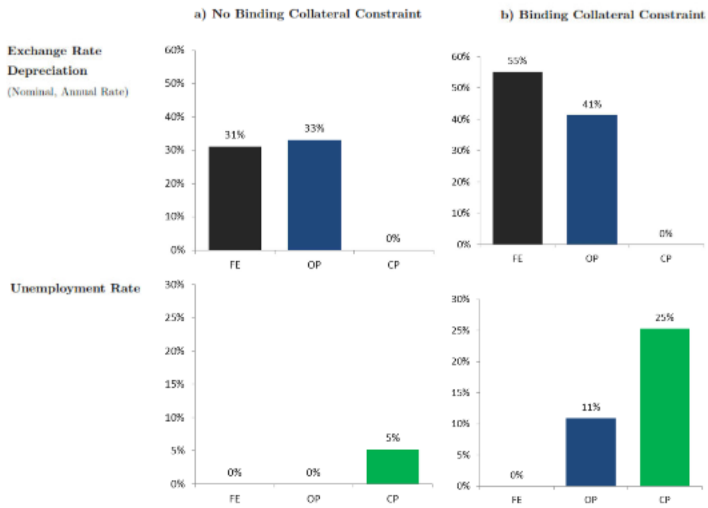
The strong quantitative similarities between the optimal and full-employment policies for the average external crisis episode are in part driven by the fact that, even during external crisis, the probability at any given quarter of being under binding collateral constraints is relatively low (3,5 percent under the optimal exchange rate policy).[9] To see this, Figure 6 depicts nominal exchange rate depreciation and unemployment during external crises divided into period in which the constraint binds and does not bind (under the optimal exchange rate policy).

When there is an external crisis but the collateral constraint does not bind (Panel a), the optimal exchange rate policy implies a very similar nominal exchange rate depreciation to the full-employment policy and features no unemployment (as shown in Proposition 2). The currency peg displays an average 5% unemployment rate as a consequence of fixing the nominal exchange rate during a negative shock. Panel b) shows the average for the quarters in which the collateral constraint does bind. Even under binding collateral constraints, optimal exchange rate policy is not to fix but to substantially depreciate the nominal exchange rate (41 percent). However, in this case, optimal exchange rate policy features less nominal exchange rate depreciation than the full-employment policy (55 percent). As a consequence of contained depreciation, a sizable involuntary unemployment appears under binding collateral constraints (11 percent). However, unemployment under

[9] The probability that a crisis episode under the optimal exchange rate policy experiences at least in one quarter binding collateral constraint is not low (36%). However, since the average duration of an episode of binding collateral constraint is only 4 quarters.

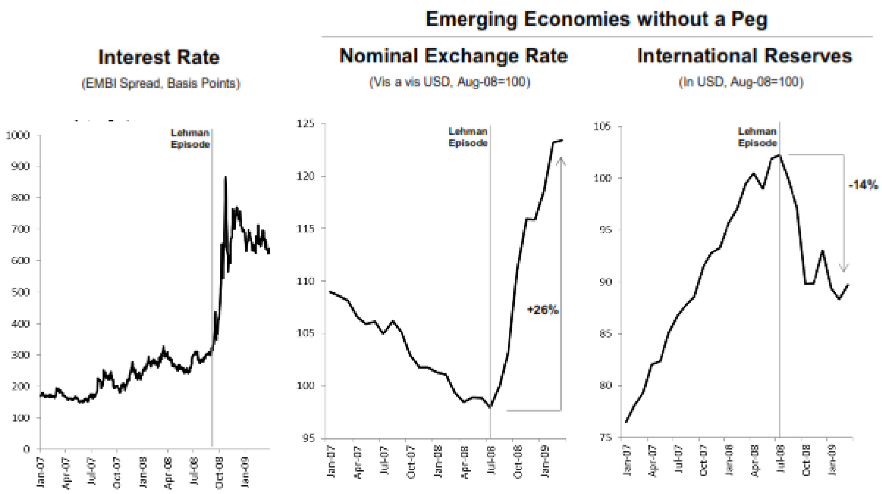
the optimal exchange rate policy is still less than half the one observed under the currency peg (25 percent).

Figure 6: Exchange Rate and Unemployment During External Crises



The large, but still contained, depreciation during periods of binding collateral constraint of the optimal policy is consistent, for instance, with the behavior observed in many emerging economies in the global financial turbulence of 2008 (see Figure 7). During this episode, emerging economies considerably depreciate the exchange rate (26% on average), but also contain the depreciation, as can be observed in the fall in international reserves. In fact, "managed-floating" regimes seem to be widely used by emerging economies (see Klein and Shambaugh (2009)).

Figure 7: Exchange Rate Policy in EMs: Lehman Financial Turmoil



5.4 Welfare Gains from the Optimal Exchange Rate Policy

We want to compare welfare under the different exchange rate policy regimes. To determine the welfare costs of an exchange rate regime i with respect to an exchange rate regime j, we compute the percentage increase in the consumption stream under exchange rate regime i that will make the representative household indifferent between that consumption stream and that under the exchange rate regime j. Formally, the compensation rate under the regime i with respect to regime j, $\lambda^{i,j}$, in a state (s_t) is implicitly defined by:

$$E\left\{\sum_{t=0}^{\infty}\beta^s\left(\frac{\left(c_{t+s}^i\left(1+\lambda^{i,j}\left(s_t\right)\right)\right)^{1-\sigma}-1}{1-\sigma}\right)\mid s_t\right\}=E\left\{\sum_{t=0}^{\infty}\beta^s\left(\frac{\left(c_{t+s}^j\right)^{1-\sigma}-1}{1-\sigma}\right)\mid s_t\right\}$$

where $i,j\in\{OP,FE,PEG\}$, $s_t=\left(y_t^T,R_t,d_t\right)$ if $i\in\{OP,FE\}$ and $s_t=\left(y_t^T,R_t,d_t,w_{t-1}\right)$ if $i=\{PEG\}$

It follows that:

$$\lambda^{i,j}\left(y_t^T,R_t,d_t\right)=\left[\frac{V^j\left(s_t\right)(1-\sigma)+(1-\beta)^{-1}}{V^i\left(s_t\right)(1-\sigma)+(1-\beta)^{-1}}\right]^{\frac{1}{1-\sigma}}-1$$

The distribution of the compensation $\lambda^{i,j}\left(s_t\right)$ depends on the distribution of the state (s_t), which in turn depends on the exchange rate regime. Thus, as noted in Schmitt-Grohe and Uribe (2011), to compute the distribution of $\lambda^{i,j}\left(s_t\right)$ one must evaluate $V^i\left(s_t\right)$ and $V^j\left(s_t\right)$ sampling from the ergodic distribution of the state under the same exchange rate regime. We choose to sample from the distribution under the full-employment exchange rate regime[10].

With this methodology, we construct the distribution of the welfare costs of the full employment policy and the currency peg with respect to optimal policy, and the distribution of welfare costs of the currency peg with respect to full-employment policy. Results are depicted in Figure 8 and Table 3. It can be seen that welfare costs of the full-employment policy are significantly lower than welfare costs of the currency peg. Moreover, we find that the full-employment regime dominates, in terms of welfare, the fixed exchange rate regime for all states. The welfare gains of the full-employment policy with respect to the

[10] The reason to sample from the distribution of the full-employment exchange rate policy is that, as shown in Section 5.2, the full-employment policy imposes a maximum level of debt which is lower than in the other two exchange rate regimes. Therefore, if we would sample, for example, from the distribution of states from the optimal policy, that would imply that for some levels of debt it would not be possible to respect the collateral constraint and non-negative consumption.

currency peg are sizable (3% of consumption per period on average). The welfare costs of the full-employment policy with respect to the optimal exchange rate policy are relatively small (0.06 percent of period consumption on average) but can reach a non trivial size (1.03 percent of consumption per period).

Figure 8: Welfare Costs by Exchange Rate Regime

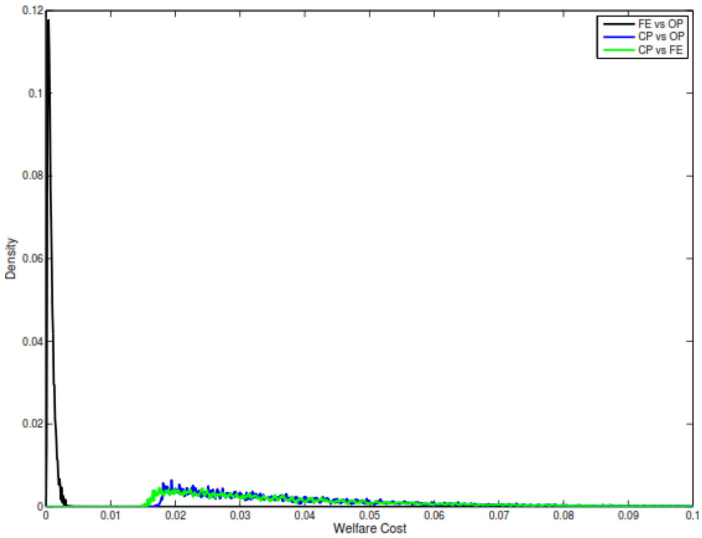
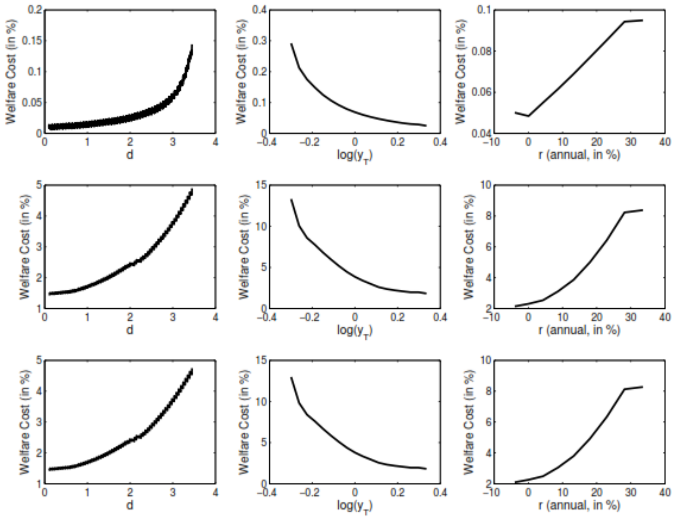


Table 3: Welfare Costs by Exchange Rate Regime

Welfare Cost of	FE	CP	CP
with respect to	OP	OP	FE
Mean	0.08	3.97	3.88
St. Dev.	0.05	2.09	2.12
Max	1.03	22.2	22.2
Min	0.01	1.56	1.32

Finally, Figure 9 depicts welfare costs as a function of the state variables. We can observe that the welfare costs of the full-employment policy and the currency peg with respect to the optimal policy are increasing in the level of debt and interest rate and decreasing in the level of tradable endowment. The welfare costs of the currency peg with respect to the full-employment follows the same pattern. This result challenges the idea that currency pegs are less harmful during periods of external crises or when external debt is high.

Figure 9: Welfare Costs by State



6 Conclusions

Combining two recent branches of the literature – downward nominal wage rigidity and the collateral constraint in the form of tradable and nontradable output – this paper provides a model economy in which exchange rate policy faces the "debt-unemployment" trade-off: the choice between reducing involuntary unemployment and relaxing the external debt limit. This trade-off captures a central discussion of the policy debate typically observed in financial crises since the Great Depression.

The first conclusion of this paper is that optimal exchange rate policy in this model economy consist of always depreciating the nominal exchange rate in response to negative shocks. If the collateral constraint does not bind, optimal exchange rate depreciation achieves full-employment. If the collateral constraint does bind, optimal exchange rate depreciation is more contained than the one that achieves full-employment. A sizable unemployment of 11% appears on average under binding collateral constraint. This highlights the role of prudential policies: to reduce the probability of hitting the collateral constraint is to reduce the probability of experiencing involuntary unemployment.

The second conclusion of this paper is that fixed exchange rate regimes are inefficient in managing the debt-unemployment trade-off, and imply sizable welfare costs. The inefficiency of currency pegs in managing the debt unemployment trade-off is given by the fact that for every level of external debt the currency peg lead to more unemployment on average and to achieve any level of unemployment the currency peg must entail less average external debt. Currency pegs do not only imply sizable welfare costs with respect to the optimal policy: a full-employment policy exchange rate regime dominates the currency peg in terms of welfare for all states. Welfare costs of currency pegs with respect to the full-employment exchange rate regime are on average 4 percent.

The last conclusion of the paper is that using an optimal capital control tax makes the optimal and full-employment exchange rate regimes very similar: the probability of experiencing a positive rate of unemployment under the optimal exchange rate policy is 0.4 percent. For this reason, the welfare costs of the full-employment policy are on average small (0.08 percent) but can achieve non-trivial levels (of 1 percent) in states with high debt and negative external conditions. These welfare costs are associated with the fact that under the full-employment exchange rate regime the economy supports less external debt.

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8 Appendices

8.1 Appendix A: Proof of Propositions

8.1.1 Proof of Proposition 1

In an equilibrium in which $\mu_t > 0$, by (CS.1) $d_{t+1.} = \overline{d}_t$. Thus, by (RC) consumption is given by $c_t^T = \overline{d_t} R_t^{-1} - d_t + y_t^T$. Replacing this expression for c_t^T and equation (MC) for c_t^N, the value function is given by $V_t = U(A(\overline{d_t} R_t^{-1} - d_t + y_t^T, F(h_t))) + \beta \mathbb{E}_t V_{t+1}$.

Then, $\left(\frac{\partial V_t}{\partial \overline{d_t}}\right)_{h_t} = R_t^{-1} U_{c,t} A_{T,t} + \beta \mathbb{E}_t V_{d,t+1} = R_t^{-1} U_{c,t} A_{T,t} - \beta \mathbb{E}_t U_{c,t+1} A_{T,t+1}$, where the last equality holds by envelope theorem. Since $\mu_t > 0$ and $\tau_t^d \geq 0$, by (H.1) it follows that $\left(\frac{\partial V_t}{\partial \overline{d_t}}\right)_{h_t} > 0$

Finally $\left(\frac{\partial V_t}{\partial h_t}\right)_{\overline{d_t}} = U_{c,t} A_{N,t} F_{h,t} > 0.$ given that $U_{c,t} > 0, A_{N,t} > 0$ and $F_{h,t} > 0.$■

8.1.2 Proof of Proposition 2

Given $\left(d_t, R_t, y_t^T\right)$, define the function$D\left(d_{t+1}, h_t\right) = d_{t+1} - \kappa\left(y_t^T + \left(\frac{1-a}{a}\right)\left(d_{t+1}R_t^{-1} - d_t + y_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{1-\frac{1}{\xi}}\right)$. Note that if $D\left(d_{t+1}, h_t\right) = 0$, the credit limit holds with equality.

Let $\left(d_{t+1}^*, \overline{d}_t^*, h_t^*\right)$ be the values for d_{t+1}, $\overline{d}_t$ and h_t in the equilibrium considered. Since equation (CC) is assumed to be binding, $d_{t+1}^* = \overline{d}_t^*$. By implicit function theorem $\frac{\partial d_{t+1}}{\partial h_t}\left(\overline{d}_t^*, h_t^*\right) = -\frac{\partial D(.)/\partial h_t}{\partial D(.)/\partial d_{t+1}}$.

First, $\partial D\left(.\right)/\partial h_t = \left(\frac{1-\xi}{\xi}\right)\kappa^N p_t$. Since $\kappa^N > 0$, in equilibrium $p_t > 0$, and by Assumption 1, $\xi < 1$ it follows that $\partial D\left(.\right)/\partial h_t > 0$.

Second, $\partial D\left(.\right)/\partial d_{t+1} = 1 - \kappa^N p_{c_t^T}(c_t^T, h_t)R_t^{-1}$. Thus, by Assumption 2, $\partial D\left(.\right)/\partial d_{t+1} > 0$.

Thus, under Assumptions 1 and 2, $\frac{\partial d_{t+1}}{\partial h_t}\left(\overline{d}_t^*, h_t^*\right)$. Finally since we are around an equilibrium with $D\left(d_{t+1}, h_t\right) = 0$, $\frac{\partial \overline{d}_t}{\partial h_t}\left(\overline{d}_t^*, h_t^*\right) = \frac{\partial d_{t+1}}{\partial h_t}\left(\overline{d}_t^*, h_t^*\right)$. ■

8.1.3 Proof of Proposition 3

Pick any $d^* < d^N$. Choose $c^{T*} = \varepsilon \in \left(0, y_L^T - \left(\frac{R_H - 1}{R_H}\right)d^*\right)$. Note that this satisfies the nonegativity constraint for consumption and that $d'^* = R_H\left(d^* + \varepsilon - y_L^T\right) < d^*$. Since for $\xi < 1$, $\frac{\partial \overline{d}_t}{\partial h} < 0$ (Remark 1), there always exist an $h < 0$ such that $d'^* \leq \kappa\left(y^T + \left(\frac{1-a}{a}\right)\right)\left(\varepsilon\right)^{\frac{1}{\xi}}\left(F\left(h\right)\right)^{1-\frac{1}{\xi}}$. It follows that $\mathcal{F}\left(d, h\right)$ is nonempty. In Figure 2, this would be equivalent to a displacement of the blue line to the right.

9 Appendix B: Omitted Constraints in Ramsey Problems

In Section 4 to define to characterize the allocation under different regimes, we follow the strategy of setting up the Ramsey problem dropping constraints. In this Appendix we show that given a solution of that Ramsey problem the omitted constraints are satisfied.

9.1 Optimal Exchange Rate Policy

To characterize the allocation under optimal exchange rate policy and optimal capital control tax in Section 4 we omitted constraints (H.1-2), (CS.1), (LM.1), (LM.3), (F.1), (G.1). Now we show that a solution $\left\{d_{t+1}^{*}, c_t^{T*}, h_t^{*}, \phi_t^{\mu*}, \phi_t^{F*}, \phi_t^{W*}\right\}$ of this problem satisfy omitted constraints given w_{t-1}.

Pick $\mu_t^* = \phi_t^{\mu*}$.This makes (CS.1) to hold: $\mu_t^* \geq 0; \mu_t^* \left(\kappa \left(y_t^T + \frac{A_N\left(c_t^{T*}, F(h_t^*)\right)}{A_T\left(c_t^{T*}, F(h_t^*)\right)} F\left(h_t^*\right)\right) - d_{t+1.}^*\right) = 0$. Pick λ_t^* to satisfy (H.2) as: $\lambda_t^* = U_c\left(c_t^{T*}, F\left(h_t^*\right)\right) A_T\left(c_t^{T*}, F\left(h_t^*\right)\right)$. Choose τ_t^{d*} to satisfy (H.1) as: $\tau_t^{d*} = 1 - R_t\beta\frac{E_t\lambda_{t+1}^* + \mu_t^*}{\lambda_t^*}$. Choose T_t^* to satisfy (G.1) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$. Pick w_t to satisfy (F.1) as: $w_t^* = \frac{A_N\left(c_t^{T*}, F(h_t^*)\right)}{A_T\left(c_t^{T*}, F(h_t^*)\right)} F'\left(h_t^*\right)$. Given w_{t-1}, pick $\in_t$ to satisfy (LM.1) with equality: $\in_t^* = \gamma\frac{w_{t-1}}{w_t^*}$. Finally, since (LM.1) holds with equality, (LM.3) always hold: $\left(w_t^* - \gamma\frac{w_{t-1}}{\in_t^*}\right)\left(\overline{h} - h_t^*\right) = 0$.

9.2 Full-Employment Exchange Rate Policy

To characterize the allocation under full-employment exchange rate policy and optimal capital control tax in Section 4 we omitted constraints (H.1-2), (CS.1), (LM.1), (LM.3), (F.1), (G.1). Now we show that a solution $\left\{d_{t+1}^{*}, c_t^{T*}, \phi_t^{\mu*}, \phi_t^{F*}, \phi_t^{W*}\right\}$ of this problem satisfy omitted constraints given w_{t-1}.

Pick $\mu_t^* = \phi_t^{\mu*}$.This makes (CS.1) to hold: $\mu_t^* \geq 0; \mu_t^* \left(\kappa \left(y_t^T + \frac{A_N\left(c_t^{T*}, F(\overline{h})\right)}{A_T\left(c_t^{T*}, F(\overline{h})\right)} F\left(\overline{h}\right)\right) - d_{t+1.}^*\right) = 0$. Pick λ_t^* to satisfy (H.2) as: $\lambda_t^* = U_c\left(c_t^{T*}, F\left(\overline{h}\right)\right) A_T\left(c_t^{T*}, F\left(\overline{h}\right)\right)$. Choose τ_t^{d*} to satisfy (H.1) as: $\tau_t^{d*} = 1 - R_t\beta\frac{E_t\lambda_{t+1}^* + \mu_t^*}{\lambda_t^*}$. Choose T_t^* to satisfy (G.1) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$. Pick w_t to satisfy (F.1) as: $w_t^* = \frac{A_N\left(c_t^{T*}, F(\overline{h})\right)}{A_T\left(c_t^{T*}, F(\overline{h})\right)} F'\left(\overline{h}\right)$. Given w_{t-1}, pick $\in_t$ to satisfy (LM.1) with equality: $\in_t^* = \gamma\frac{w_{t-1}}{w_t^*}$. (FE) implies that (LM.2) is always satisfied. Finally, since (LM.1) and (LM.2) hold with equality, (LM.3) always hold: $\left(w_t^* - \gamma\frac{w_{t-1}}{\in_t^*}\right)\left(\overline{h} - \overline{h}\right) = 0$.

9.3 Fixed Exchange Rate Policy

To characterize the allocation under fixed exchange rate policy and optimal capital control tax in Section 4 we omitted constraints (H.1-2), (CS.1), (LM.3), and then show that the solution satisfies these constraints. The Lagrangian of the Ramsey problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} U\left(A\left(c_t^T, F\left(h_t\right)\right)\right) + \\ +\phi_t^F\left[\frac{d_{t+1}}{R_t} - d_t + y_t^T - c_t^T\right] + \\ +\phi_t^\mu\left[\kappa\left(y_t^T + \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{1-\frac{1}{\xi}}\right) - d_{t+1}\right] + \\ +\phi_t^W\left[\overline{h} - h_t\right] \\ +\phi_t^R\left[w_t - \gamma w_{t-1}\right] \\ +\phi_t^P\left[\left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{-\frac{1}{\xi}} F'\left(h_t\right) - w_t\right] \end{array} \right\}$$

The optimality condition associated to this problem are (RC), (CC), (LM.2), the first order conditions

$$\begin{aligned}
\frac{\phi_t^F}{R_t} &= \beta E_t \phi_{t+1}^F + \phi_t^\mu \\
\phi_t^F &= U_{c,t} A_{T,t} + \phi_t^\mu \kappa \left(\frac{1}{\xi}\right)\left(\frac{1-a}{a}\right)\left(\frac{c_t^T}{F\left(h_t\right)}\right)^{\frac{1}{\xi}-1} + \phi_t^P \left[\frac{1}{\xi}\left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}-1} F\left(h_t\right)^{-\frac{1}{\xi}} F'\left(h_t\right)\right] \\
\phi_t^W &= F_{h,t}\left[U_{c,t} A_{N,t} + \phi_t^\mu \left(\frac{\xi-1}{\xi}\right)\kappa\left(\frac{1-a}{a}\right)\left(\frac{c_t^T}{F\left(h_t\right)}\right)^{\frac{1}{\xi}}\right] + \\
&\quad + \phi_t^P \left[-\frac{1}{\xi} F\left(h_t\right)^{-\frac{1}{\xi}-1} F'\left(h_t\right) + F\left(h_t\right)^{-\frac{1}{\xi}} F''\left(h_t\right)\right] \\
\phi_t^R &= \beta\gamma E_t \phi_{t+1}^R + \phi_t^P \qquad \text{(FOC.CP.1-4)}
\end{aligned}$$

and the complementary slackness conditions

$$\begin{aligned}
\phi_t^\mu &\geq 0; \phi_t^\mu \left[\kappa\left(y_t^T + \left(\frac{1-a}{a}\right)\left(c_t^T\right)^{\frac{1}{\xi}} F\left(h_t\right)^{1-\frac{1}{\xi}}\right) - d_{t+1}\right] = 0 \\
\phi_t^W &\geq 0; \phi_t^W \left[\overline{h} - h_t\right] = 0 \\
\phi_t^R &\geq 0; \phi_t^R \left[w_t - \gamma w_{t-1}\right] = 0 \qquad \text{(CS.CP.1-3)}
\end{aligned}$$

Pick $\mu_t^* = \phi_t^{\mu *}$.This makes (CS.1) to hold: $\mu_t^* \geq 0; \mu_t^* \left(\kappa\left(y_t^T + \frac{A_N\left(c_t^{T*}, F\left(\overline{h}\right)\right)}{A_T\left(c_t^{T*}, F\left(\overline{h}\right)\right)} F\left(\overline{h}\right)\right) - d_{t+1.}^*\right) = 0$. Pick λ_t^* to satisfy (H.2) as: $\lambda_t^* = U_c\left(c_t^{T*}, F\left(\overline{h}\right)\right) A_T\left(c_t^{T*}, F\left(\overline{h}\right)\right)$. Choose τ_t^{d*} to satisfy (H.1) as: $\tau_t^{d*} = 1 - R_t \beta \frac{E_t \lambda_{t+1}^* + \mu_t^*}{\lambda_t^*}$. Choose T_t^* to satisfy (G.1) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$. Finally, we can show that (LM.3) always hold by contradiction (as in Schmitt-Grohe and Uribe, 2012). Suppose that in the Ramsey allocation (LM.3) does not hold for some date and state. By CS.CP.2-3 this implies that $\phi_t^W = \phi_t^R = 0$. By FOC.CP.3 this means that $\phi_t^P > 0$.But this would contradict condition FOC.CP.4 (since $\phi_t^W \geq 0$).