# Accounting for idiosyncratic wage risk over the business cycle

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## Outline

- Stylized facts
  - idiosyncratic wage risk is countercyclical
  - largely driven by the cyclicality of the *wage variance conditional on unemployment*, and a *composition* effect
- Structural links
  - model with OTJ search gives reasonably good account of this connection
  - cyclicality of reservation wage is key

## Why should we care?

- wages are the primary source of income for a large fraction of people
- incomplete insurance markets
- cyclically concentrated idiosyncratic shocks can increase the welfare cost of business cycles significantly

▸ Related literature

## Why wage volatility?

- wage volatility: cross-sectional *variance* of *innovations* to log *wages*
- *wages*, because they are often taken to be the exogenous source of risk in macro models
- *innovations*, because growth rates are a plausible indicator of the size of unanticipated and uninsurable shocks to individuals
- *variance*, because it allows an intuitive decomposition
- there are other important moments (notably persistence) which we do not address

## Data and methodology

- data are from PSID, 1967–1992 details
- given that we are interested in second moments, sample size is relatively small, especially after we partition the data
- we use a multilevel Bayesian model, in order to get sharper estimates and to account for uncertainty after decomposition

# Statistical model for stylized facts

$$
\begin{aligned}
\mathrm{d}w_{i,t} &\sim \mathrm{N}\left( X_{i,t}\,\theta + \alpha_t\,,\, \sigma_t^2 \right) \\
\alpha_t &\sim \mathrm{N}\left( Z_t\,\kappa, \varsigma_\alpha^2 \right) \\
\sigma_t^2 &\sim \mathrm{N}\left( Z_t\,\eta\,, \varsigma_{\sigma^2}^2 \right), \quad \sigma_t^2 > 0.
\end{aligned}
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- $\mathrm{d}w_{i,t}$ is change in individual $i$'s log wage, at time $t$
- $X_{i,t}$ are individual-specific covariates (age, education)
- $\alpha_t$ is the year-specific mean
- $\sigma_t^2$ is the year-specific variance ("wage volatility")
- $Z_t$ are covariates for each year (constant & unemp. rate)
- the relations between $Z$ and the $\alpha$'s and $\sigma^2$'s have error variances $\varsigma_\alpha^2$ and $\varsigma_{\sigma^2}^2$

multilevel statistical model | reference priors

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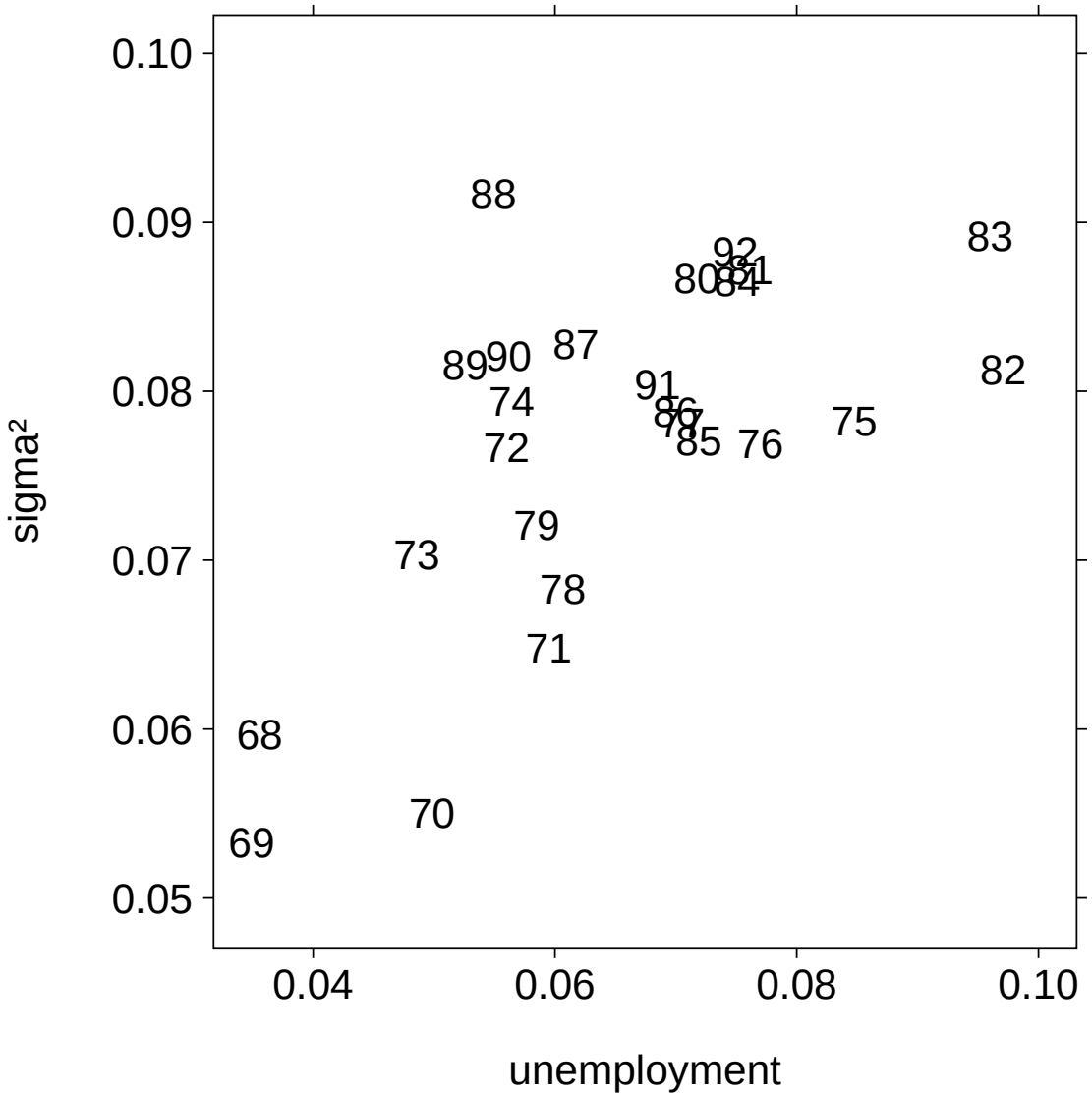
# Wage volatility versus the unemployment rate




posterior means

robustness quantiles $\alpha_t$

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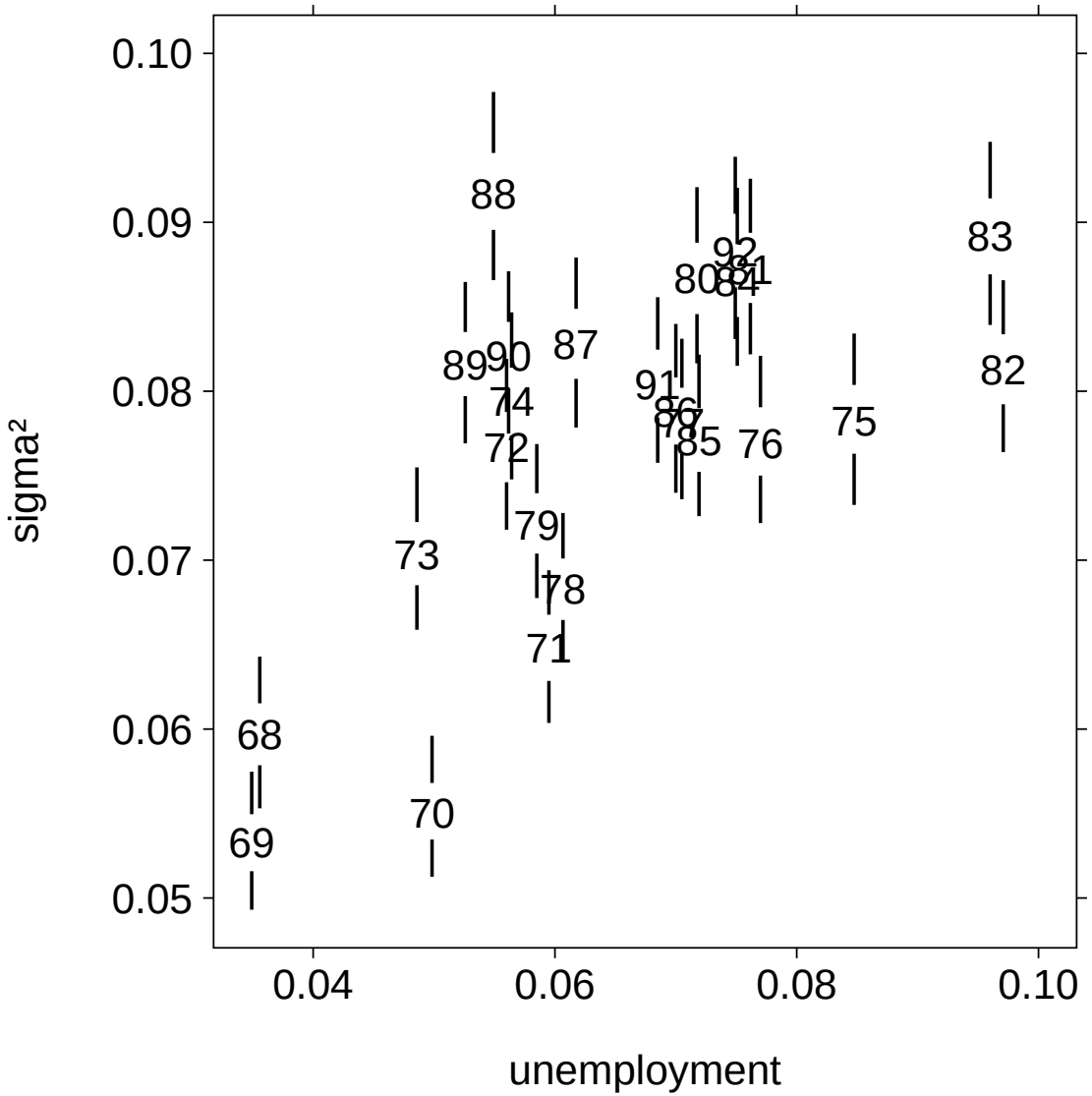


posterior means
+ error bars (5–25–75–95%)

robustness quantiles $\alpha_t$

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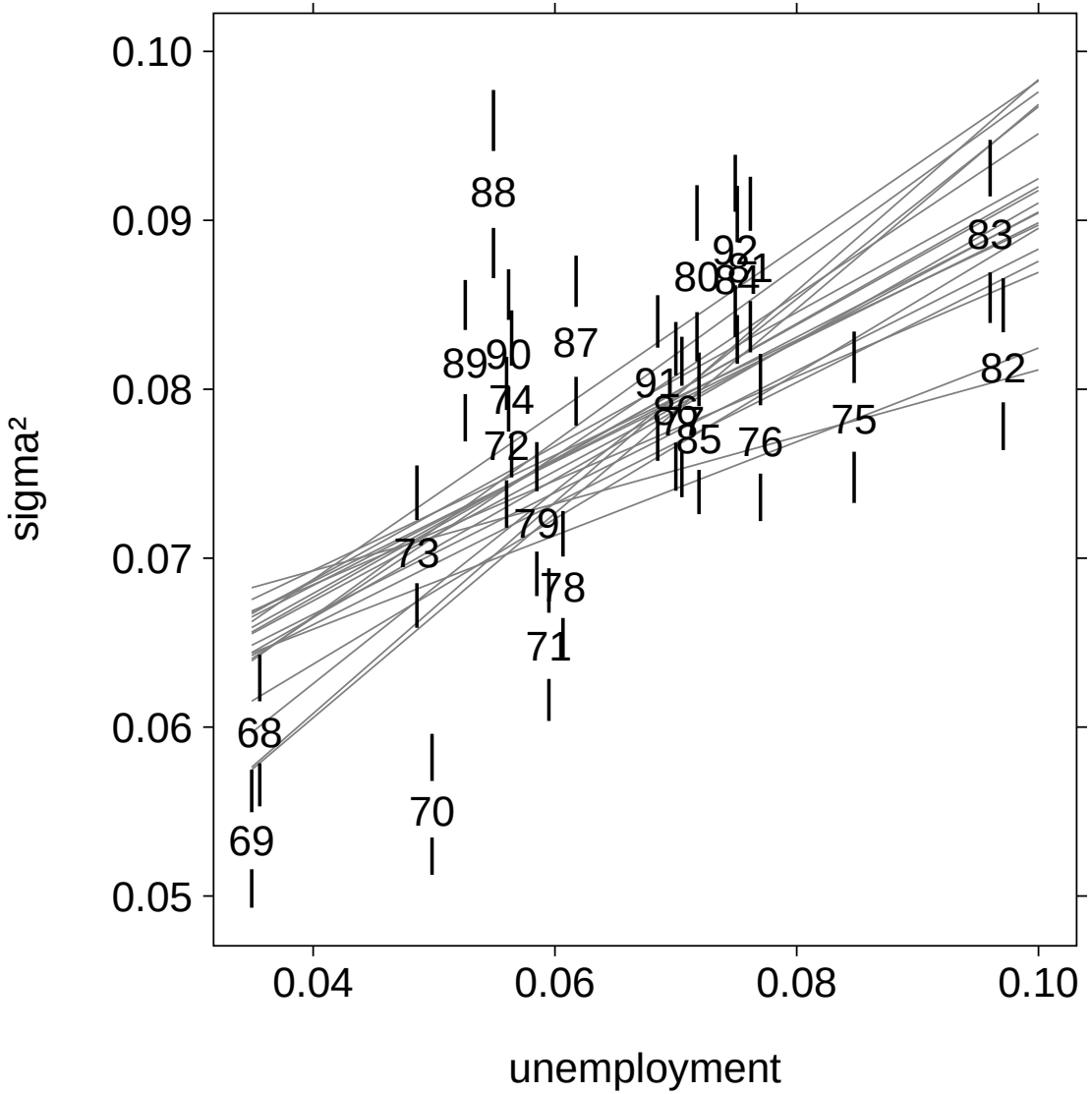



posterior means
+ error bars (5–25–75–95%)
+ multilevel regression lines

regression slope posterior mean is 0.42

robustness quantiles $\alpha_t$

# Conditioning on unemployment

## Motivation

- Unemployment shocks are associated with losses of firm-specific human capital and changes in match-specific productivities.
- Unemployment has a substantial and variable effect on wages at all times.
- Greater incidence of unemployment could potentially explain increased wage risk.

## Implementation: partitioning the sample

- no unemployment hours in year $t$ or $t-1 \Rightarrow$ **E**mployment group
- otherwise **U**nemployment group
- **U** is 15–20% of the sample.

# Connection between aggregate and subgroup wage volatility: decomposing the regression slope

- cyclical changes from the composition (more unemployment in recessions)
- cyclical changes conditional on subgroups

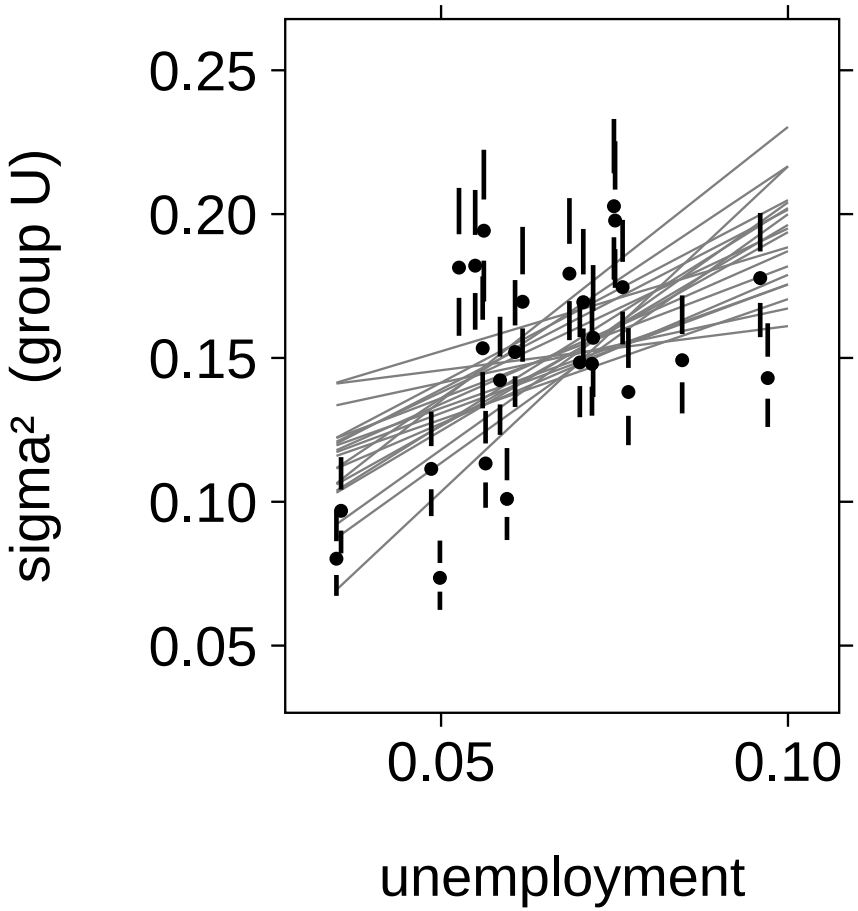
$$\frac{\partial \sigma_t^2}{\partial u_t} = \underbrace{\bar{\sigma}_U^2 \frac{\partial \pi_U}{\partial u_t} + \bar{\sigma}_E^2 \frac{\partial \pi_E}{\partial u_t}}_{\text{composition effect}} + \underbrace{\bar{\pi}_U \frac{\partial \sigma_{t,U}^2}{\partial u_t} + \bar{\pi}_E \frac{\partial \sigma_{t,E}^2}{\partial u_t}}_{\text{within-group effect}}$$

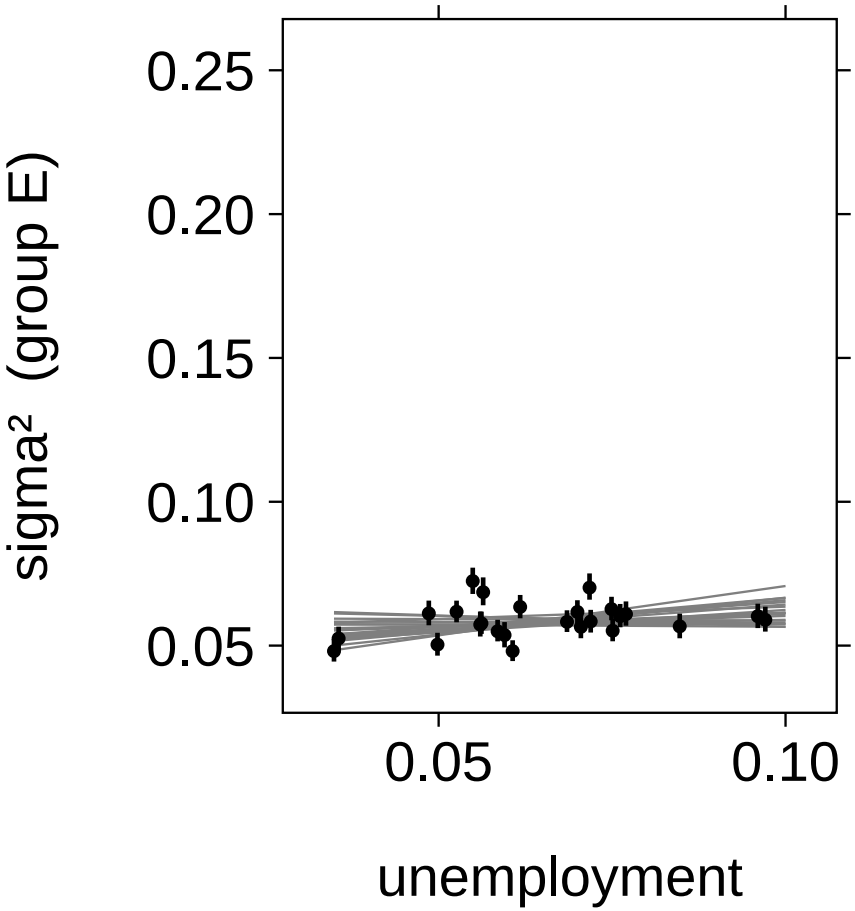
$$+ \text{second order terms}$$

$$+ \text{between effect}$$

- Extend statistical model to group means and variances.
- Apply decomposition to draws form posterior.

# Estimates for the two groups



## Decomposing the regression slope of wage volatility on the unemployment rate (posterior mean 0.42)

| | mean | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|
| $d\sigma_U^2$ | 0.2361 | 0.1836 | 0.2147 | 0.2361 | 0.2587 | 0.2900 |
| $d\sigma_E^2$ | 0.0775 | 0.0308 | 0.0574 | 0.0772 | 0.0971 | 0.1244 |
| composition | 0.1040 | 0.0987 | 0.1018 | 0.1039 | 0.1060 | 0.1095 |
| between | 0.0074 | 0.0014 | 0.0043 | 0.0070 | 0.0100 | 0.0147 |
| error | 0.0018 | −0.0075 | −0.0019 | 0.0016 | 0.0057 | 0.0113 |

## Normalized decomposition (sums to 1)

| | mean | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|
| $d\sigma_U^2$ | 0.5533 | 0.4732 | 0.5206 | 0.5539 | 0.5866 | 0.6320 |
| $d\sigma_E^2$ | 0.1791 | 0.0810 | 0.1404 | 0.1809 | 0.2193 | 0.2722 |
| composition | 0.2464 | 0.2060 | 0.2269 | 0.2433 | 0.2626 | 0.2952 |
| between | 0.0174 | 0.0033 | 0.0103 | 0.0165 | 0.0236 | 0.0352 |
| error | 0.0038 | −0.0186 | −0.0045 | 0.0039 | 0.0131 | 0.0248 |

(last two digits)

# Search-and-matching frictions

## Question

Can time-series dynamics of job-offer and separation rates generate observed patterns of wage risk?

## Partial equilibrium search model

- only source of cyclical variation: job-offer and separation rates
- these are exogenous, we sidestep the unemployment volatility puzzle
- keep wage offer distribution and other parameters constant

## Outline

1. summary of model with aggregate dynamics
2. calibration and results
3. intuition about sensitivity from steady state

# Wage-ladder model

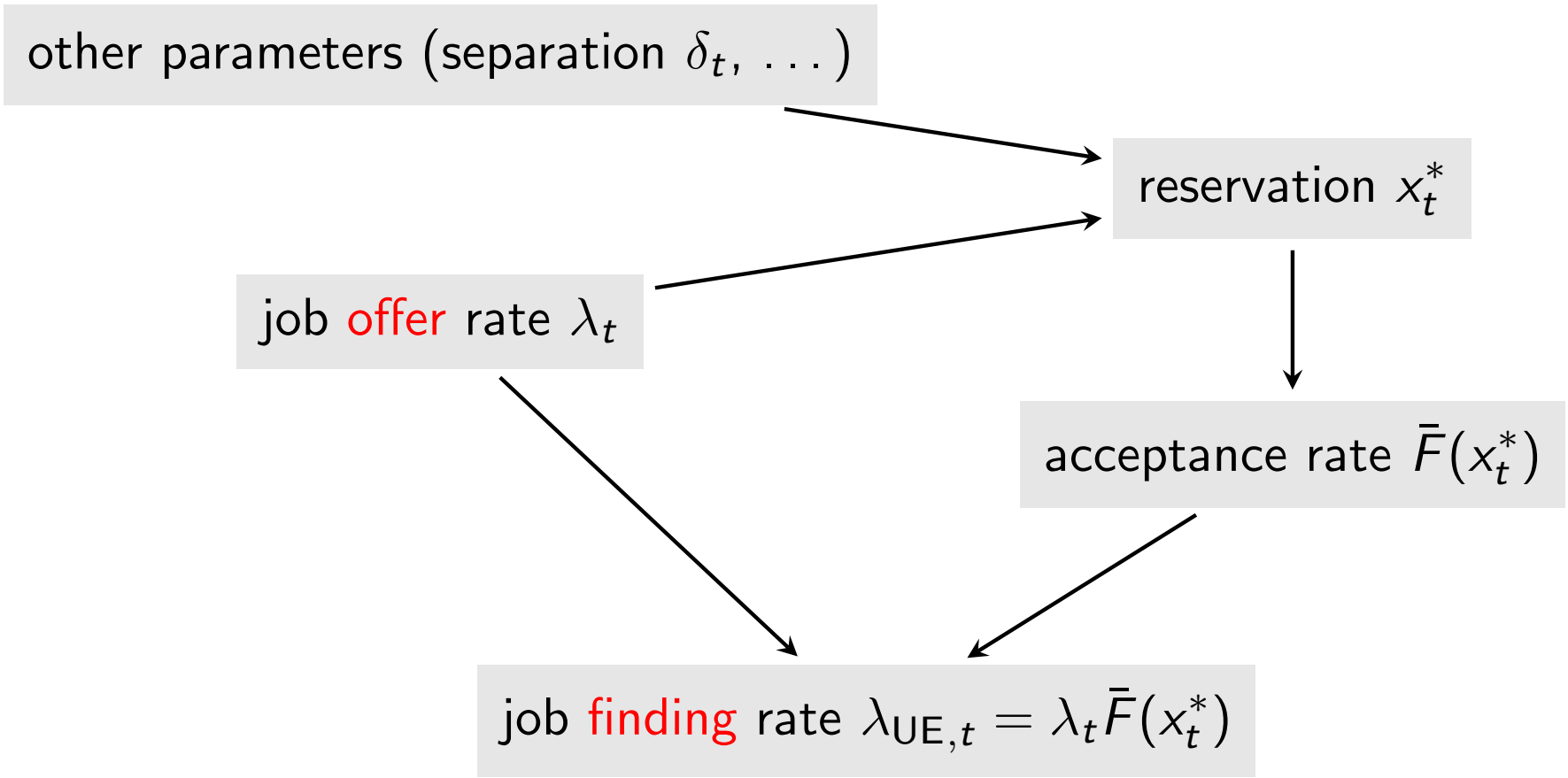
- log utility, discrete time, discounting with $\beta = 1/(1+r)$
- log wage of employed individual $i$ at time $t$

$$w_{i,t} = p_t + a_{i,t} + x_{i,t}.$$

- $p_t$ is an aggregate component in wages
- $a_{i,t}$ is individual's persistent productivity (across matches)
- $x_{i,t}$: match component, drawn from $F$, constant within a match
- unemployed
  - offers $x_{i,t} \sim F$ arrive with probability $\lambda_t$
  - (log) unemployment benefit $p_t + a_{i,t} + b$
- employed
  - offers $x_{i,t} \sim F$ arrive with probability $\gamma\lambda_t$
  - exogenous separation with probability $\delta_t$
- $p_t, a_{i,t}, x_{i,t}$ independent
  $\Rightarrow$ linear separation, only $x_{i,t}$ influences decisions

# Mechanics of the model

- employed search on the job, move up the wage ladder
- unemployed have a reservation $x_t^*$, depends on labor market conditions



# Calibration of time invariant parameters

- time unit is $1/48$ year $\approx$ one week
- relative OTJ search efficiency $\gamma = 0.151$, using job-to-job flows and distribution-independent calibration of Nagypál (2005)
- $F$ normal with mean 0 (normalization), standard deviation $s = 0.337$ chosen to match $\text{Var}(dw_{i,t})$ in the data
- log unemployment benefit $b = \log(0.17)$ chosen so that $F(x^*) = 0.56$
  - actual cutoff has little effect on results distribution
  - generated wage dispersion *per se* is important
  - mean-min ratio result of Hornstein, Krusell and Violante (2012)
- individual-specific productivity process $a_{i,t}$
  - AR(1) process, calibrated from stayers, matching moments
  - autoregressive coefficient: 0.997, standard deviation for innovation 0.000385, standard deviation for measurement error 0.148

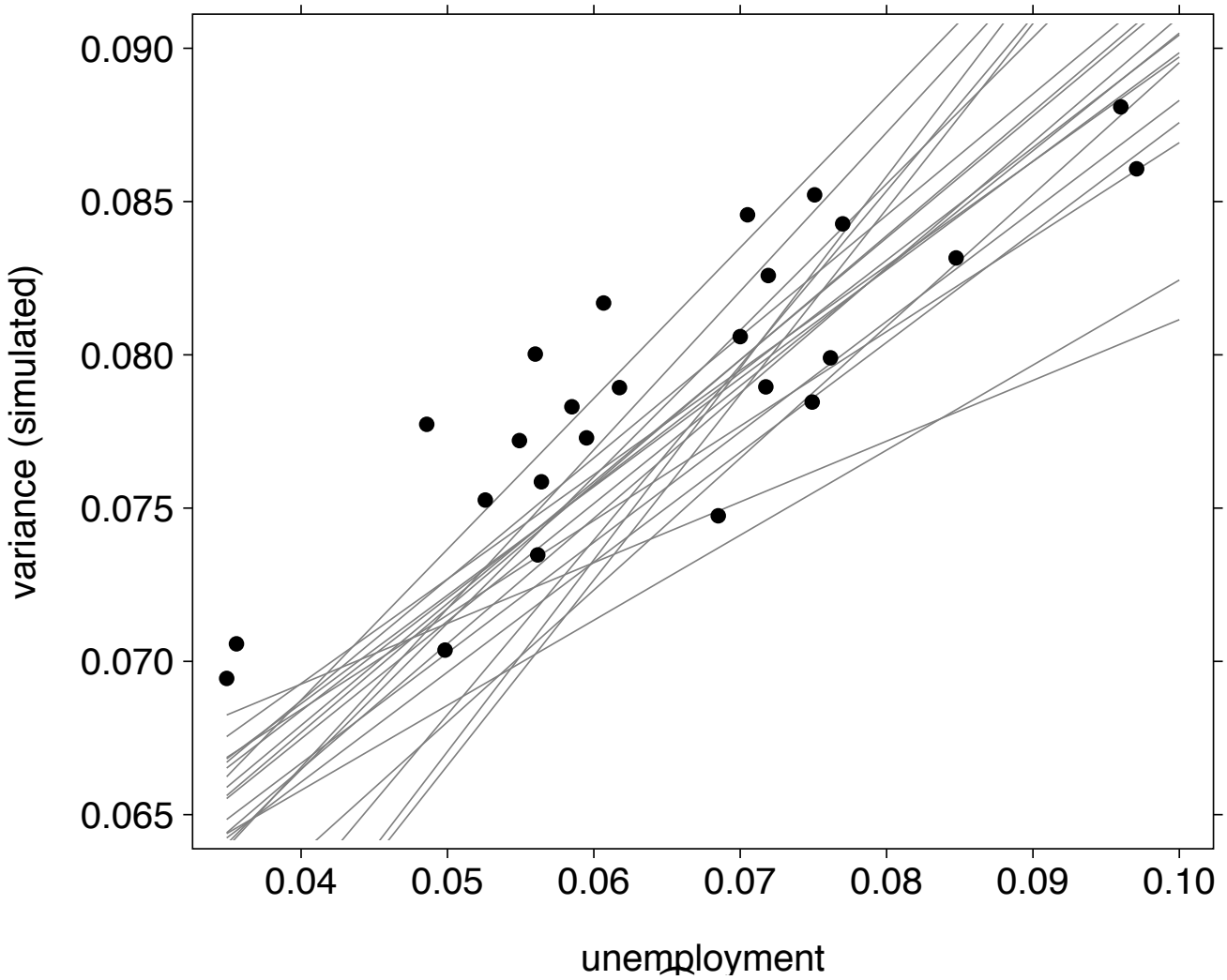
## Calibration of the time series job offer and separation rates

- separation rate (exogenous) from Shimer (2007)
- job offer rate (exogenous) chosen so that the resulting job finding rate (endogenous) matches data from Shimer (2007)

## Quantitative experiment

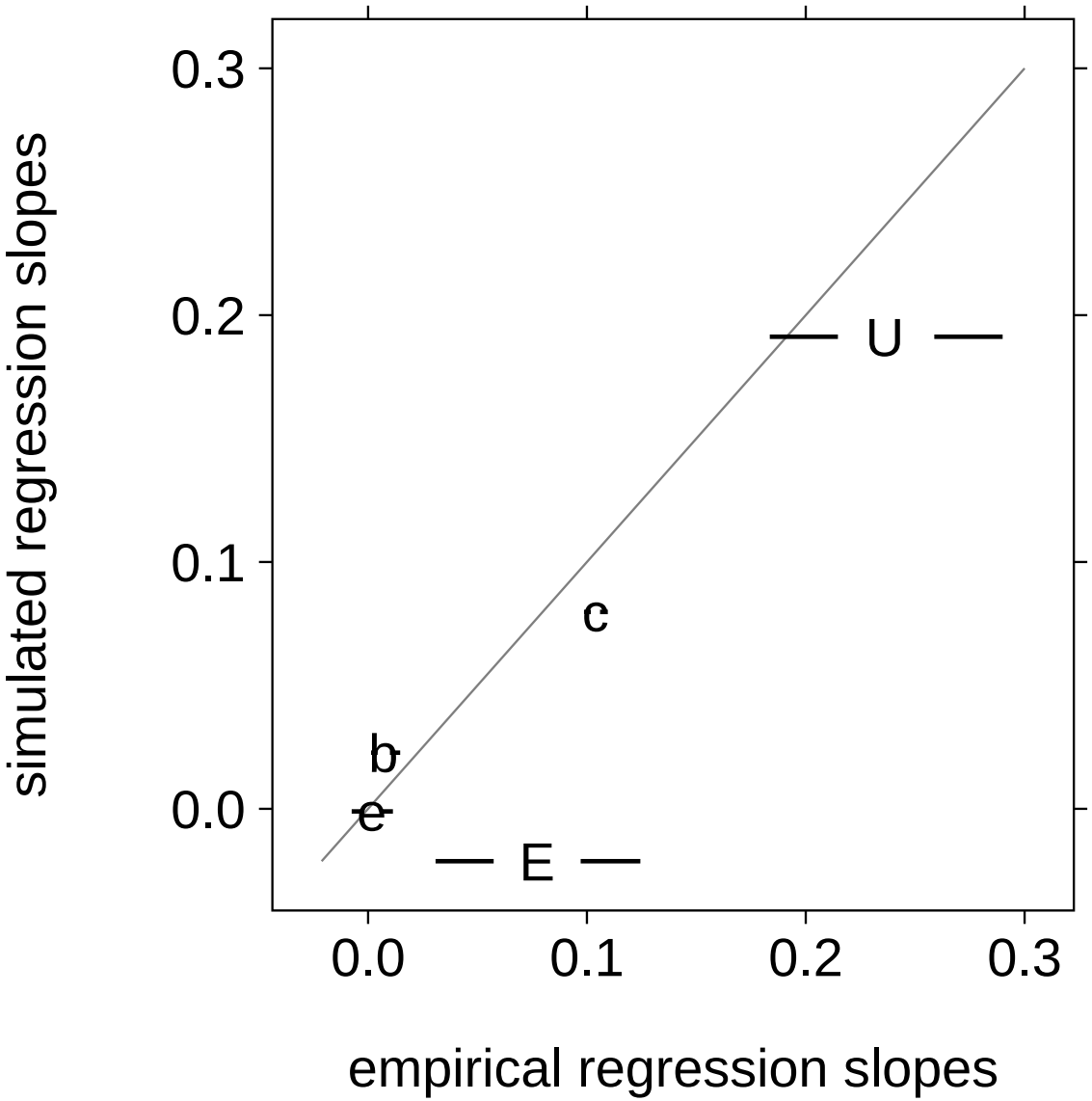
- calibrate the model
- simulate wages from 1948 to 1992 (1967 - 1992 is PSID sample period, $t < 1967$ is burn-in period)
- simulate at weekly frequency and aggregate to years as in data
- calculate and decompose cyclical variation in wage risk (repeating empirical exercise)

# Results: wage volatility vs the unemployment rate




- Regression slope is 0.27 (cf. 0.42 from empirical results)

# Decomposition: simulated vs empirical




Legend:

- cyclicality of **U**nemployment and **E**employment group wage volatility
- **c**omposition
- **b**etween-group effect
- **e**rror

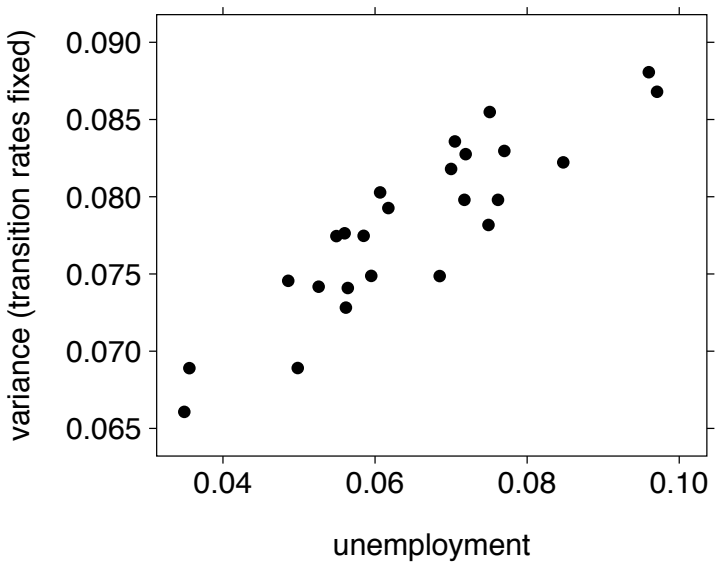
means

# Fixing transition rates and reservation $x^*$

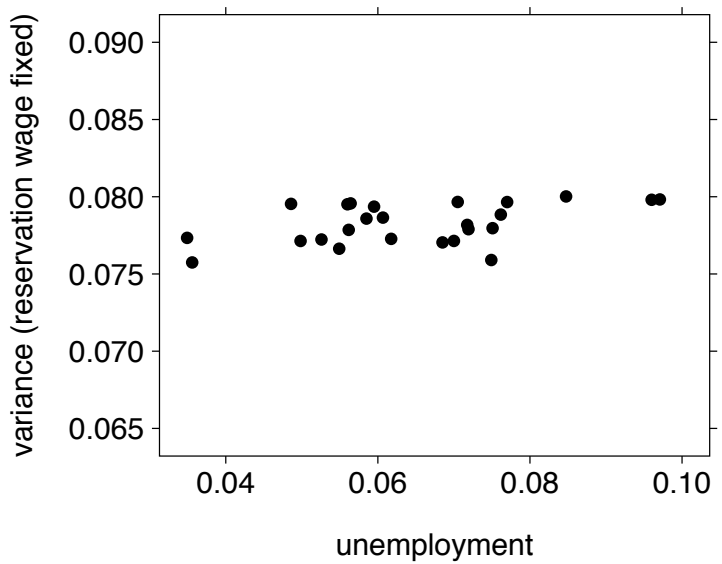
## Two channels for impact on wage risk

- direct effect of transition probabilities,
- indirect effect through changes in reservation wage.

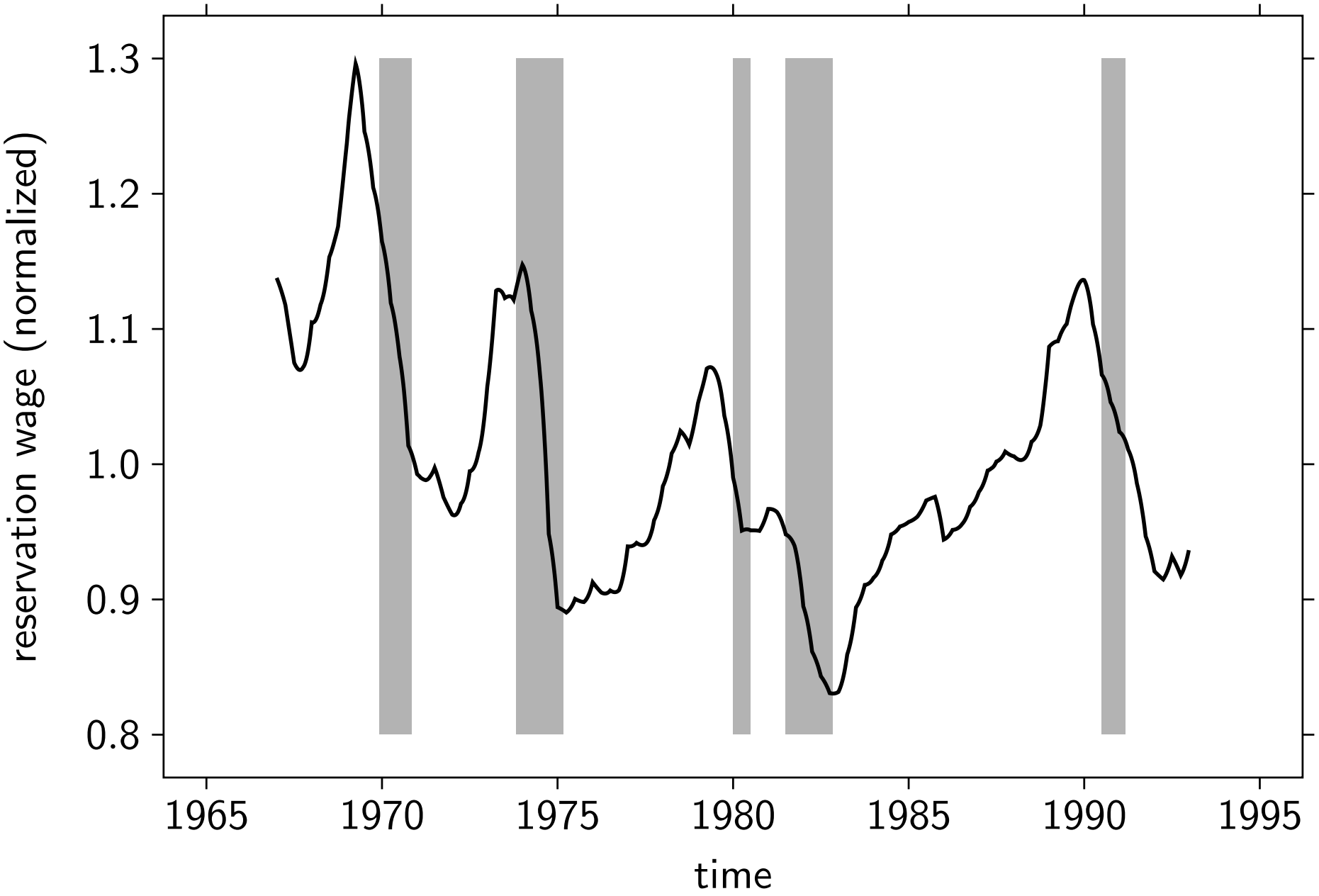
variation in reservation wage




variation in transition rates



# Movements in the reservation wage



# Sensitivity to the offer distribution

Ignore on-the-job search for algebraic simplicity:

$$x^* = b + \underbrace{\frac{\lambda}{r+\delta}}_{\equiv \chi} \int_{x^*}^{\infty} (x - x^*) dF(x^*)$$

Total derivative:

$$dx^* = d\chi \frac{x^* - b}{\chi} - dx^* \chi \int_{x^*}^{\infty} dF(x^*)$$

or equivalently, independently of the distribution $F$.

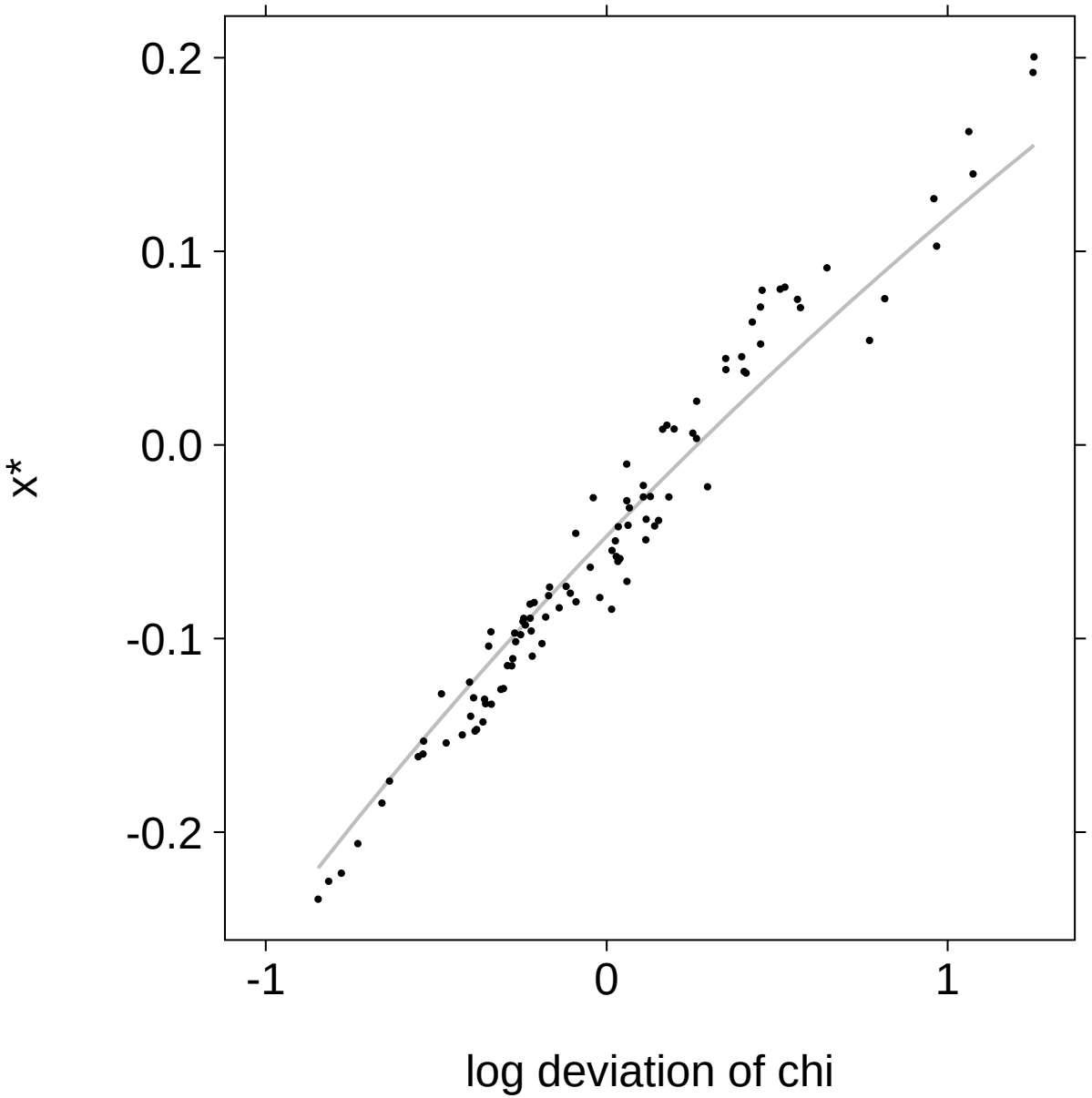
$$\frac{dx^*}{d \log \chi} = \frac{x^* - b}{1 + \chi \bar{F}(x^*)}$$

With on-the-job search, $\chi$ is also a sufficient statistic, and the elasticity is

$$\frac{dx^*}{d \log \chi} = \frac{x^* - b}{1 + \chi \bar{F}(x^*)} \left(1 - \gamma \left(1 + \chi \bar{F}(x^*) \, \mathrm{M}(F, x^*)\right)\right)$$

where $\mathrm{M}(F, x^*) \in [0, 1]$ is related to the "shape" of the offer distribution, for a normal distribution it just depends on the quantile of $x^*$ details

# Steady state vs. dynamic model




- dots: dynamic model
- line: steady state comparative statics

# Summary and conclusion

## Empirical investigation of the sources of cyclical variation in wage risk

- countercyclical variance of wage changes
- mostly among those experiencing unemployment
- smaller role for a composition effect

## Calibrated model with OTJ search: good quantitative account

| | empirical | simulated |
|---|---|---|
| regression slope | 0.42 | 0.27 |
| variance within unemployment group | 0.24 | 0.19 |
| composition effect | 0.10 | 0.08 |

## Future work

- persistence of wage shocks
- slope for the **E** group — cyclicality of relative search efficiency $\gamma$?
- realistic unemployment benefit/duration calibration

# Related literature

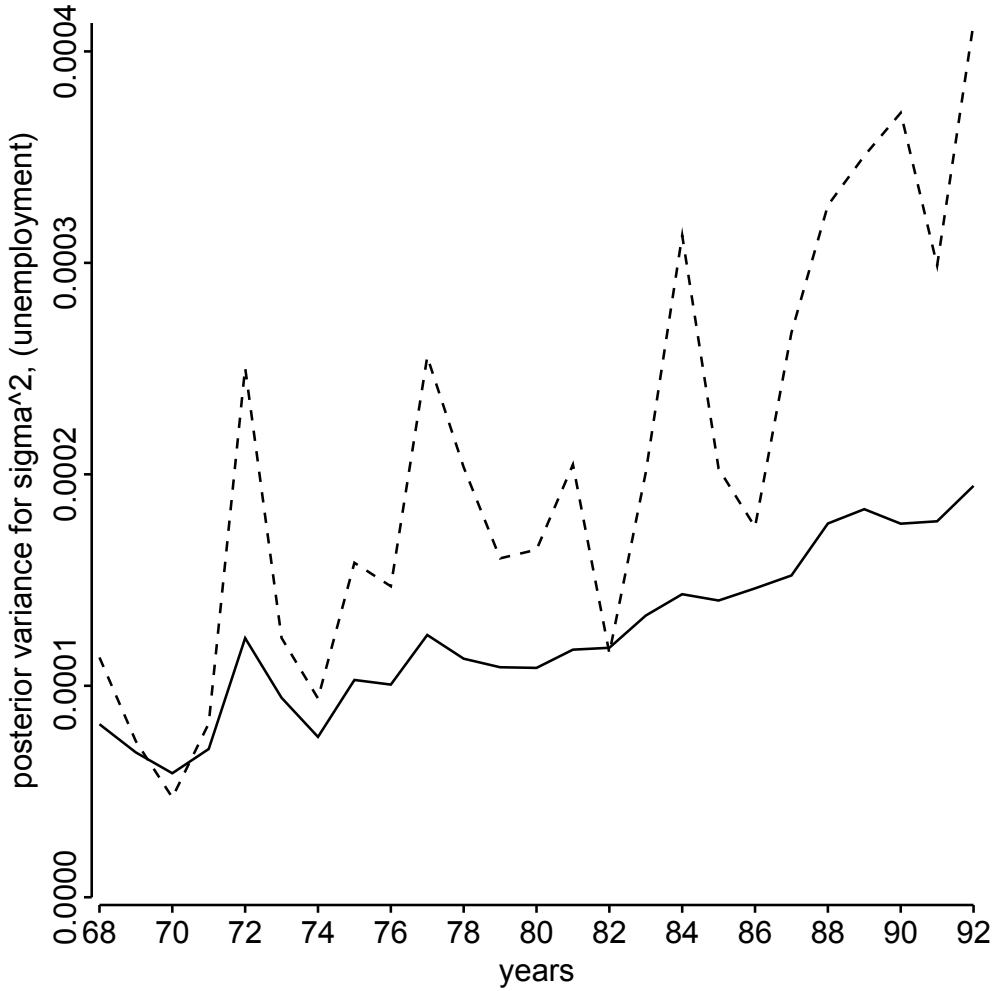
- Cyclical variation in income risk
  - Storesletten et al. (2004).
- Welfare cost of business cycles with heterogeneous households
  - Imrohoruglu (1989), Atkeson and Phelan (1994), Krusell and Smith (1999), Beaudry and Pages (2001), Storesletten et al. (2001); Krebs (2003, 2007); De Santis (2007); Krusell et al. (2009).
- Business cycle models with incomplete markets and search frictions
  - Gomes et al. (2001), Costain and Reiter (2005, 2007), Shao and Silos (2007), Rudanko (2009, 2011), Nakajima (2010), Bils et al. (2011), Krusell et al. (2010), Jung and Kuester (2011).
- Steady state implications of search frictions for wage dynamics
  - Postel-Vinay and Turon (2010).

# Why use a multilevel model?

- Estimate all equations jointly.
- As opposed to using a two-stage regression and point estimates, posterior uncertainty about $\eta$ reflects uncertainty about the $\sigma_t^2$.
- Sharper estimates of $\sigma_t^2$'s: estimation pools information to the extent supported by the data. example

# Posterior variance of estimates

Posterior variance of $\sigma_t^2$ parameter in the unemployment group with and without multi-level pooling.



# Priors and estimation

- non-informative reference priors

$$p(\theta) \propto 1$$
$$p(\kappa) \propto 1$$
$$p(\eta) \propto 1$$

- sufficient sample size at each level of the model to make the posterior distribution proper.
- half-Cauchy priors for variances

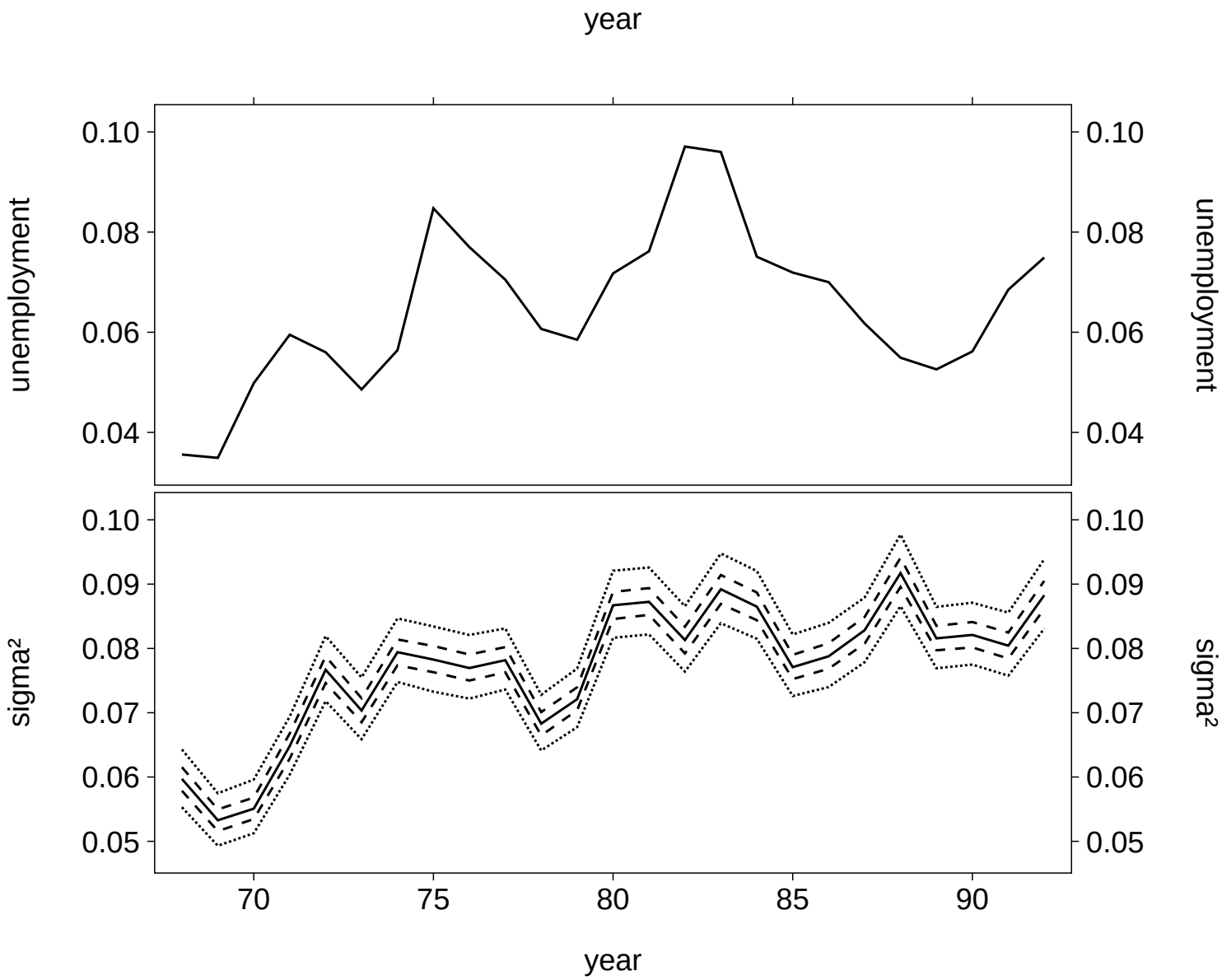
$$p(\varsigma_\alpha) \propto (\varsigma_\alpha^2 + s_\alpha^2)^{-1}$$
$$p(\varsigma_{\sigma^2}) \propto (\varsigma_{\sigma^2}^2 + s_{\sigma^2}^2)^{-1}.$$

  with $s_\alpha = s_{\sigma^2} = 1$.
- Gibbs sampler (many steps are sampling linear regressions)

# Data

- Panel Study of Income Dynamics (PSID) SRC sample.
- income from 1967 to 1992
- male heads between ages 25 and 60
- students, business owners, self-employed individuals excluded
- only individual who worked at least 320 hours ($\approx$ 8 weeks) in previous and current years.
- deflate using CPI-Research index
- $\text{wage} = \dfrac{\text{annual earnings}}{\text{annual hours}}$
- first difference of log wages

# Trend vs cycle?



# Trend vs cycle?

*A look at high-frequency movements:*

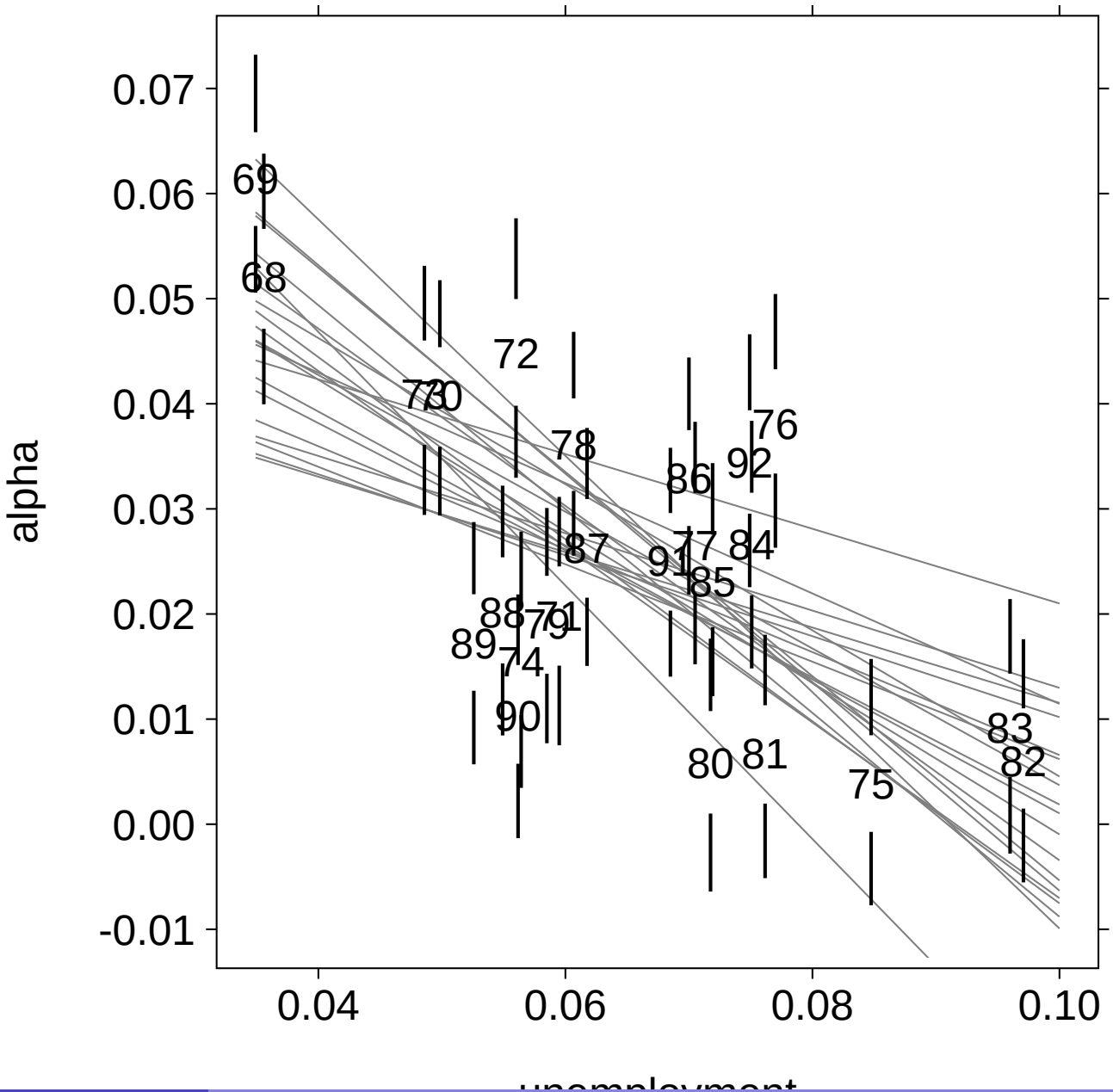
- During recessions, unemployment rate rises quickly and forces behind other secular trends are most likely more or less stable.
- Do the increases in wage risk during recessions reflect our estimated relationship?
- Identify peaks and troughs with NBER dates.
- Compute $\Delta\sigma_t^2/\Delta u_t$ where $\Delta$ is difference between trough and peak.
- Result should be close to estimate of $\eta_{\text{unemp}} = 0.42$.

| mean | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|
| 0.31 | 0.19 | 0.26 | 0.31 | 0.36 | 0.43 |

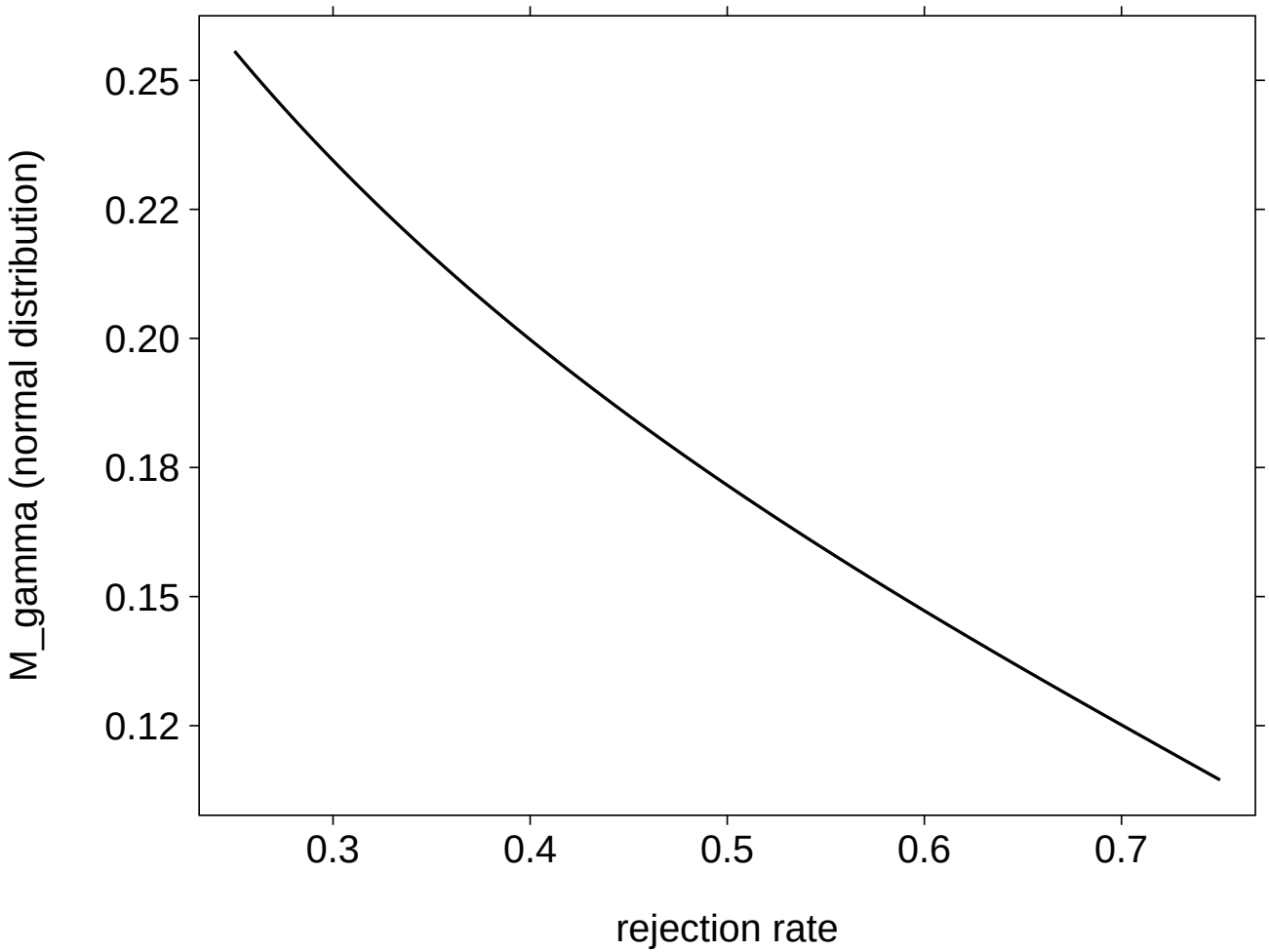
# Posterior quantiles

| | mean | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|
| $\theta_{\text{age}}$ | −0.0014 | −0.0016 | −0.0015 | −0.0013 | −0.0012 | −0.0011 |
| $\theta_{\text{edu}}$ | 0.0028 | 0.0019 | 0.0024 | 0.0028 | 0.0032 | 0.0038 |
| $\kappa_{\text{const}}$ | 0.0258 | 0.0202 | 0.0236 | 0.0257 | 0.0279 | 0.0313 |
| $\kappa_{\text{unemp}}$ | −0.6402 | −1.0127 | −0.7858 | −0.6368 | −0.4904 | −0.2911 |
| $\varsigma_{\alpha}$ | 0.0145 | 0.0101 | 0.0123 | 0.0141 | 0.0163 | 0.0201 |
| $\eta_{\text{const}}$ | 0.0771 | 0.0741 | 0.0758 | 0.0771 | 0.0784 | 0.0802 |
| $\eta_{\text{unemp}}$ | 0.4255 | 0.2257 | 0.3428 | 0.4244 | 0.5077 | 0.6311 |
| $\varsigma_{\sigma^2}$ | 0.0091 | 0.0068 | 0.0079 | 0.0089 | 0.0099 | 0.0119 |

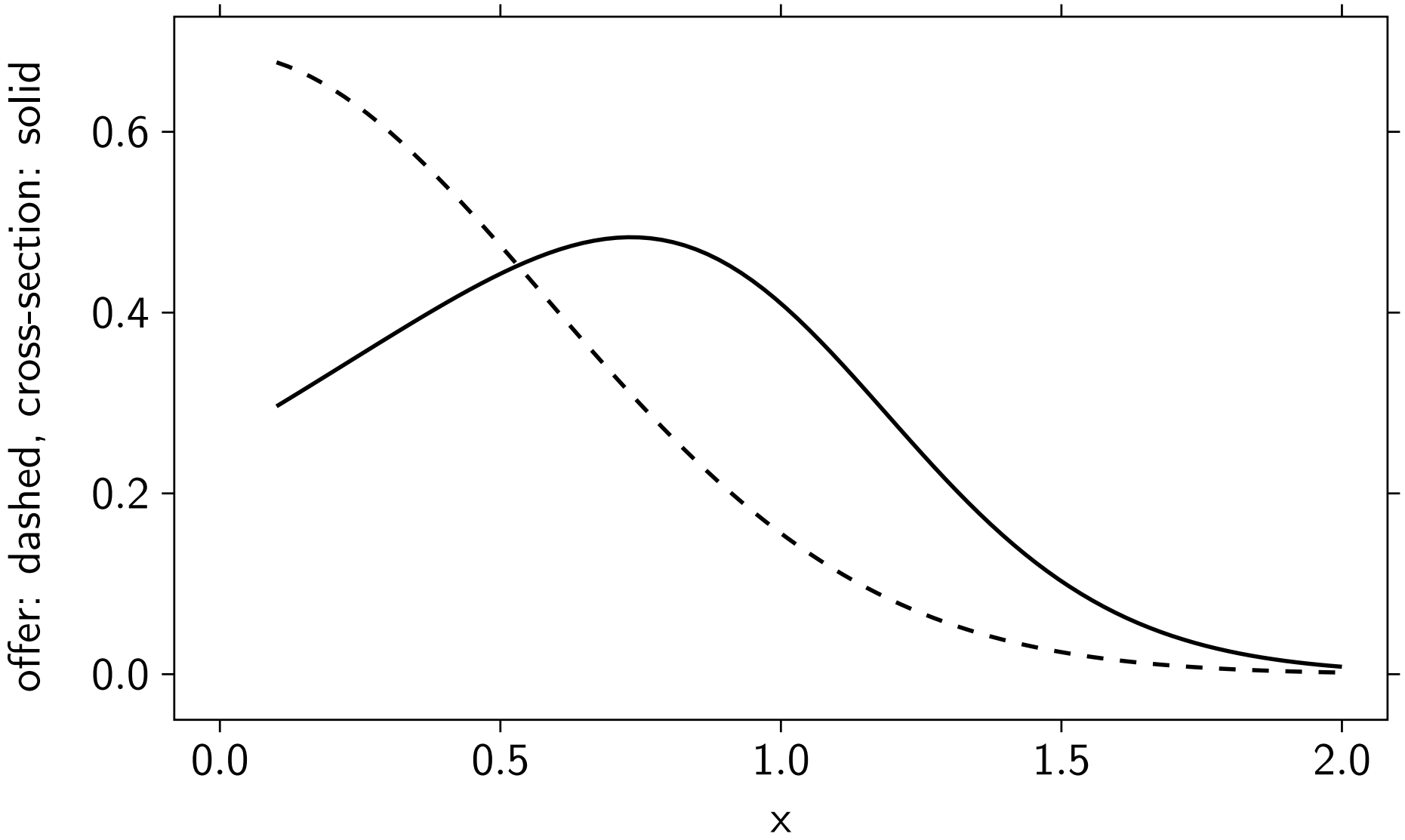
# $\alpha_t$ versus the unemployment rate



# $\mathrm{M}(F, x^*)$ for normal distribution



# Steady state cross-sectional distribution




$\log 90/50 \approx 0.52$, $\log 50/10 = 0.51$

# Sanity check: cyclical variation in mean wage changes

- slope coefficients for regression of mean wage change on unemployment rate
- no changes in $p_t$, everything comes from match-specific component $x_{i,t}$

| Group | Data | Model |
|---|---|---|
| Unemployment | -1.13 | -1.09 |
| No unemployment | -0.44 | -0.39 |