When does a central bank’s balance sheet require fiscal support?

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Disclaimer: The views expressed are ours and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System
Yet another balance sheet simulation?

• Hall and Reis (2013), Carpenter et al. (2013), Greenlaw et al. (2013) examine likely scenarios, based on historically normal behavior of interest rates and demand for central bank liabilities.

• We study these scenarios, but also look for unlikely, scenarios in which failure of fiscal-monetary coordination or excessive size of the central bank balance sheet could create a conflict between maintaining stable inflation and maintaining central bank solvency.

• We think that recent history teaches that considering unlikely, but disastrous behavior of asset markets, so as to be prepared if they occur and to reduce their likelihood, is worthwhile.
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- We think that recent history teaches that considering unlikely, but disastrous behavior of asset markets, so as to be prepared if they occur and to reduce their likelihood, is worthwhile.

- We look at complete, though simplified, economic model in order to study why a central bank’s balance sheet matters at all and the consequences of a lack of fiscal backing for the central bank.
Contribution

- Interest rates, inflation, and seigniorage are endogenous, hence we can answer questions such as:

1. Under what conditions does the central bank need fiscal support taking future seigniorage into account?
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2. How does the sensitivity of the CB balance sheet to shocks (changes in real rate, inflation expectations,..) depend on interest rate policy? And is therefore conventional policy constrained by “solvency” issues?
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2. How does the sensitivity of the CB balance sheet to shocks (changes in real rate, inflation expectations,..) depend on interest rate policy? And is therefore conventional policy constrained by “solvency” issues?

3. How does the size of the central bank balance sheet affect the potential for its needing direct fiscal support?
The model

- Simple, perfect-foresight, non-linear model.
  - Exogenous real interest rate $\rho$ and income $Y$
  - Flexible prices
Households

- Households maximize

\[ \int_0^\infty e^{\beta} \log C_t \, dt \]

subject to

\[ C_t (1 + \psi(v_t)) + \dot{F}_t + \dot{V}_t + \dot{M}_t + q_t \dot{B}^P \frac{B^P}{P^t} \]

\[ = Y + \rho_t F_t + r_t \frac{V_t}{P^t} + (\chi + \delta - q_t \delta) \frac{B^P}{P} - \tau_t. \]
Households

- Households maximize
  \[
  \int_0^\infty e^{\beta \log C_t} dt
  \]
  subject to

  \[
  C_t(1 + \psi(v_t)) + \dot{F}_t + \frac{\dot{V}_t + \dot{M}_t + q_t \dot{B}^P}{P_t} = Y + \rho_t F_t + r_t \frac{V_t}{P_t} + (\chi + \delta - q_t \delta) \frac{B^P}{P} - \tau_t.
  \]

- \( V \): overnight reserves. \( B \): Woodford (2001)-style bond (depreciates at rates \( \delta \), coupon \( \delta + \chi \), duration \( \delta^{-1} \))
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- \( V \): overnight reserves. \( B \): Woodford (2001)-style bond (depreciates at rates \( \delta \), coupon \( \delta + \chi \), duration \( \delta^{-1} \))

- The model used in simulations has population growth \( n \) and productivity growth \( \gamma \).
Transactions technology

\[ v_t = \frac{P_t C_t}{M_t} \]

\[ \psi(v) = \frac{\psi_0 v}{1 + \psi_1 v} \]

- \( \psi_1 < 0 \Rightarrow v_t^{-1} \) asymptotes to \( \psi_1^{-1} \) as \( r_t \to \infty \)

- \( \psi_1 > 0 \Rightarrow \) Implies real balances go to zero when nominal rate reaches \( \psi_0 / \psi_1^2 \).
Fiscal policy

- Financial Authority budget constraint:

\[ g_t + (\chi + \delta - \delta q_t) \frac{B}{P} = \tau_t + \tau^C_t + q_t \frac{\dot{B}}{P_t} \]

- \( \tau^C_t \) remittances from CB. \( B = B^P + B^C \).
Fiscal policy

• Financial Authority budget constraint:

\[ g_t + (\chi + \delta - \delta q_t) \frac{B}{P} = \tau_t + \tau^C_t + q_t \frac{\dot{B}}{P_t} \]

• \( \tau^C_t \) remittances from CB. \( B = B^P + B^C \).

• Passive fiscal policy (except for possibly switching to active under explosive paths):

\[ \tau_t = \xi_0 + \xi_1 q \frac{B}{P} \]
Conventional and unconventional monetary policy

- Reaction function for Interest on reserves:

\[
\dot{r} = \theta_r \cdot \left( \bar{r} + \theta_\pi \left( \frac{\dot{P}}{P} - \bar{\pi} \right) - r \right)
\]
Conventional and unconventional monetary policy

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  \[ \dot{r} = \theta_r \cdot \left( \bar{r} + \theta_\pi \left( \frac{\dot{P}}{P} - \bar{\pi} \right) - r \right) \]

- Unconventional monetary policy: path for $B^C$ (Treasuries held by CB)
Central bank budget constraint and rule for remittances

- The central bank budget constraint is:

\[ q_t \frac{\dot{B}_t}{P_t} - \frac{\dot{V}_t + \dot{M}_t}{P_t} = (\chi + \delta - \delta q_t) \frac{B_t}{P} - r_t \frac{V_t}{P} - \tau_t \]

This approximates the idea that the Fed holds bonds to maturity and does not mark to market, while using a rule that tries to keep bank capital \( K = qB/P - V/P \) constant.
Central bank budget constraint and rule for remittances

- The central bank budget constraint is:

\[ q_t \frac{\dot{B}_C}{P_t} - \frac{\dot{V}_t + \dot{M}_t}{P_t} = (\chi + \delta - \delta q_t) \frac{B_C}{P} - r_t \frac{V_t}{P_t} - \tau_t^C \]

- Rule for remittances:

\[ \tau_t^C = \max \left\{ 0, \chi \frac{B_C}{P_t} - r_t \frac{V_t}{P_t} - \bar{r}K \right\} . \]

This approximates the idea that the Fed holds bonds to maturity and does not mark to market, while using a rule that tries to keep bank capital \( K = qB_C / P - V \) constant.
A present value take on CB solvency and remittances

\[ q \frac{B_0^C}{P_0} - \frac{V_0}{P_0} + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt \]

Mkt value of assts - reserves

PDV seigniorage

\[ = \int_0^\infty C_t e^{-\int_0^t \rho_s ds} dt \]

PDV remittances

• PDV of remittances does not depend on:
  1. Future path of \( B^C \) (whether \( B^C \) is held to maturity or not)
  2. Accounting rules, e.g., rule for remittances, deferred assets...

• Time path of \( C_t \) may (but does not have to...)

Del Negro, Sims  
Central bank's balance sheet
A present value take on CB solvency and remittances

\[
q \frac{B_0^C}{P_0} - \frac{V_0}{P_0} + \left( \int_0^{\infty} \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt \right)
\]

Mkt value of assts - reserves \quad PDV seigniorage

\[
= \int_0^{\infty} \tau_t^C e^{-\int_0^t \rho_s ds} dt
\]

PDV remittances

- PDV of remittances does not depend on:
  1. Future path of \( B^C \) (whether \( B^C \) is held to maturity or not)
  2. Accounting rules, e.g., rule for remittances, deferred assets ...

- Time path of \( \tau^C \) may (but does not have to...)
Four levels of CB balance sheet problems

Level 1  Accounting capital (book value of $B^C - V - M$) $< 0$. Or rule for remittances implies $\tau^C = 0$ for some period.

- No fiscal support ($\tau^C < 0$) required. In fact, PDV of $\tau^C$ may actually increase.
Four levels of CB balance sheet problems

Level 1  Accounting capital (book value of $B^C - V - M$) < 0. Or rule for remittances implies $\tau^C = 0$ for some period.

- No fiscal support ($\tau^C < 0$) required. In fact, PDV of $\tau^C$ may actually increase.

Level 2  $qB^C - V < 0$

- No fiscal support required as long as PDV seignorage
  \[
  \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt > V - qB^C
  \]
- Depends on $q$ (future path of $r$, premia) but also on size of $V$ (whether households trade $M$ for $V$)
- The higher duration $\delta^{-1}$, the more likely this situation is to arise.
- The likelihood likely depends on conventional monetary policy.
If inflation gets out of control, $r$ becomes very high, $\frac{M}{P}$ may ↓ → $\int_{0}^{\infty} \frac{\dot{M}_t}{P_t} e^{-\int_{0}^{t} \rho_s ds} dt$ ↓

- Fiscal support required ($\int_{0}^{\infty} \tau_t^C e^{-\int_{0}^{t} \rho_s ds} dt \rightarrow \tau_t^C < 0$ for some $t$). Fiscal policy may continue to be passive
Level 3  \( qB^C - V + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt < 0 \)

- If inflation gets out of control, \( r \) becomes very high, \( \frac{M}{P} \) may \( \downarrow \rightarrow \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt \downarrow \)

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Level 4  Explosive paths/hyperinflation

- These paths cannot be ruled out by a monetary authority with no fiscal powers.
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Level 4 Explosive paths/hyperinflation

- These paths cannot be ruled out by a monetary authority with no fiscal powers.
- The CB balance sheet is relevant only to whether the CB can sustain the explosive path until whatever triggers a fiscal intervention occurs. With high inflation, real balances and therefore seignorage may drop toward zero.
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Level 4 Explosive paths/hyperinflation

- These paths cannot be ruled out by a monetary authority with no fiscal powers.
- The CB balance sheet is relevant only to whether the CB can sustain the explosive path until whatever triggers a fiscal intervention occurs. With high inflation, real balances and therefore seignorage may drop toward zero.
- Why not FTPL on CB budget constraint? Seignorage is a kind of tax, but its base disappears as the value of cash shrinks, so it cannot provide the real anchor required by a FTPL argument.
Solving the model

- Marginal utility
  
  \[ c^{-1} = \lambda \left( 1 + \psi(v) + v \psi'(v) \right) \]

- Euler eq.

  \[ \beta - \rho = \frac{\dot{\lambda}}{\lambda} \]

- Fisher eq.

  \[ r - \frac{\dot{P}}{P} = \rho \]

- Money demand

  \[ v^2 \psi'(v) = r \]
• No arbitrage (long term bonds)

\[
\frac{\chi + \delta}{q} - \delta + \frac{\dot{q}}{q} = r
\]

\[
\rightarrow q_0 = (\chi + \delta) \int_0^\infty e^{-\left(\int_0^t r_s ds + \delta t\right)} dt
\]
• No arbitrage (long term bonds)

$$\frac{\chi + \delta}{q} - \delta + \frac{\dot{q}}{q} = r$$

$$\Rightarrow q_0 = (\chi + \delta) \int_0^\infty e^{-\left(\int_0^t r_s ds + \delta t\right)} dt$$

• Fisher + Taylor rule:

$$r_t = \int_0^\infty e^{-\left(\theta \pi - 1\right)r_s \theta \pi} (\rho_{t+s} + \pi_{t+s}) ds + \frac{\bar{r} - \theta \pi \bar{\pi}}{\theta \pi - 1} + \kappa e^{\left(\theta \pi - 1\right)r_t} ds$$
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\[\rightarrow q_0 = (\chi + \delta) \int_0^\infty e^{-\left(\int_0^t r_s ds + \delta t\right)} dt\]

• Fisher + Taylor rule:

\[
r_t = \int_0^\infty e^{-\left(\theta \pi - 1\right)\theta r_s \theta \pi (\rho_{t+s} + \chi_{t+s})} ds + \frac{\bar{r} - \theta \pi \bar{\pi}}{\theta \pi - 1} + \kappa e^{(\theta \pi - 1)\theta r_t} ds
\]

• Resource constraint:

\[
C_0 \left(\int_0^\infty (1 + \psi(v)) e^{-\int_0^t (\rho_s - \dot{\xi}) ds} dt\right) = F_0 + (Y - g) \int_0^\infty e^{-\int_0^t \rho_s ds} dt
\]
Results from a simple steady-state change

\[ \theta_{\pi} = 2.00, \ \rho = 0.01, \ \bar{\rho} = 0.01, \ \beta = 0.01, \ \psi_0 = 0.01, \ \psi_1 = 0.10, \]
\[ \theta_r = 0.60, \ F_0 = 2.00, \ V_0 = 3.00, \ B^C_0 = 4.00. \]

**Table:** Effect of 3% inflation scare

<table>
<thead>
<tr>
<th></th>
<th>base</th>
<th>after shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>.01</td>
<td>0.07</td>
</tr>
<tr>
<td>( \nu )</td>
<td>1.11</td>
<td>3.60</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td>( m_0 )</td>
<td>.909</td>
<td>0.276</td>
</tr>
<tr>
<td>( dp )</td>
<td>.0500</td>
<td></td>
</tr>
<tr>
<td>( dm )</td>
<td></td>
<td>-1.1911</td>
</tr>
<tr>
<td>( dM )</td>
<td></td>
<td>-1.1411</td>
</tr>
<tr>
<td>( dpvs )</td>
<td>0</td>
<td>3.0000</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Duration</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>2.5</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Direct proportional capital loss from ( r ) change</td>
<td>-0.15</td>
<td>-0.29</td>
<td>-0.5455</td>
<td>-1.0000</td>
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</table>

Funding gap to be filled with \( dpvs \)

<p>| | | | | |</p>
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<tbody>
<tr>
<td></td>
<td>0.5983</td>
<td>1.7823</td>
<td>3.4190</td>
<td>5.2602</td>
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</table>
Parameters

normalization, foreign assets
\[ Y - g = 1 \quad F_0 = 0 \]

initial assets, reserves, and currency (t=0 is Dec 31, 2012)
\[ B^c/P = .237 \quad V/P = .132 \]
\[ M/P = .108 \]

discount rate, reversion to st.st., population and productivity growth
\[ \beta = 0.01 \quad \gamma = 0.0075 \]
\[ \varphi_1 = 0.750 \quad n = 0.0075 \]

monetary policy
\[ \theta_\pi = 2 \quad \theta_r = 1 \]
\[ \bar{\pi} = 0.02 \]

money demand
\[ \psi_0 = 0.00001 \quad \psi_1 = -0.05 \]
(match \( M/P \) at \( r = .0025 \), \( \psi_{ss} = 0.00056 \))

bonds: duration and coupon
\[ \delta^{-1} = 6 \quad \chi = 0.035 \]
A scatter plot of short term interest rates and M/PC

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Central bank's balance sheet
Baseline and scenarios

- Baseline: real rate path chosen so that $r$ path roughly matches Carpenter et al.

- Scenarios (time 0 “surprise”, perfect foresight afterwards)
  1. “Higher rates” Carpenter et al. scenario
  2. 10 year “inflation scare”
  3. Explosive paths (hyperinflations)
Baseline and “Higher Rates” Scenario

Nominal Short Rate

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>r</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
<td>4</td>
<td>4.5</td>
<td>5</td>
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Baseline and “Higher Rates” Scenario
## Balance Sheet Implications

<table>
<thead>
<tr>
<th></th>
<th>qB/P</th>
<th>P</th>
<th>seig. (1)+(2)</th>
<th>τC</th>
<th>q</th>
<th>ΔM/P</th>
<th>B/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Baseline scenario</td>
<td>0.132</td>
<td>0.949</td>
<td>1.081</td>
<td>0.0024</td>
<td>1.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Higher rates (β)</td>
<td>0.120</td>
<td>0.136</td>
<td>0.256</td>
<td>0.0028</td>
<td>1.09</td>
<td>-0.007</td>
<td>14.02</td>
</tr>
<tr>
<td>(3) Higher rates (γ)</td>
<td>0.132</td>
<td>1.238</td>
<td>1.369</td>
<td>0.0030</td>
<td>1.09</td>
<td>0.004</td>
<td>70.71</td>
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Seignorage and M/PC

Data

Model
Inflation Scare – different $\theta_{\pi}$’s

- Fisher equation:
  \[ r_t = \rho_t + \frac{\dot{P}_t}{P_t} + x \]
  for $t \in [0, 10]$, where $x = P(\Delta P) \times \Delta P$

Nominal Short Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>Baseline</th>
<th>$\theta_{\pi} = 2$</th>
<th>$\theta_{\pi} = 3$</th>
<th>$\theta_{\pi} = 1.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td></td>
<td>4.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td>5.2</td>
<td>5.3</td>
<td>5.2</td>
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<tr>
<td>2014</td>
<td></td>
<td>6.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>2015</td>
<td></td>
<td>7.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>2016</td>
<td></td>
<td>8.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>2017</td>
<td></td>
<td>9.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>
## Balance Sheet Implications / Inflation Scare

<table>
<thead>
<tr>
<th></th>
<th>$qB/P$</th>
<th>$-V/P$</th>
<th>PDV seig.</th>
<th>$(1)+(2)$</th>
<th>$\bar{\tau}^C$</th>
<th>$q$</th>
<th>$\Delta M/P$</th>
<th>$\bar{B}/B$</th>
</tr>
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<td>1.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Inflation scare</td>
<td>0.041</td>
<td>0.951</td>
<td>0.993</td>
<td>0.0022</td>
<td>0.94</td>
<td>-0.049</td>
<td>7.05</td>
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<tr>
<td>Higher $\theta_\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(6) Inflation scare</td>
<td>0.053</td>
<td>0.905</td>
<td>0.958</td>
<td>0.0021</td>
<td>0.98</td>
<td>-0.048</td>
<td>8.25</td>
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<tr>
<td>Lower $\theta_\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(8) Inflation scare</td>
<td>0.012</td>
<td>0.966</td>
<td>0.977</td>
<td>0.0023</td>
<td>0.81</td>
<td>-0.047</td>
<td>4.85</td>
<td></td>
</tr>
</tbody>
</table>
Explosive paths – different $\theta_\pi$’s

- Explosive paths: $r_t = \text{stable solution} + \kappa e^{\theta r (\theta_\pi - 1) t}$

![Graph showing explosive paths with different $\theta_\pi$ values from 2012 to 2017]
## Balance Sheet Implications / Explosive path

<table>
<thead>
<tr>
<th></th>
<th>(1) $qB/P$</th>
<th>(2) PDV seig.</th>
<th>(3) $(1)+(2)$</th>
<th>(4) $\tau^C$</th>
<th>(5) $q$</th>
<th>(6) $\Delta M/P$</th>
<th>(7) $\bar{B}/B$</th>
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<tbody>
<tr>
<td>Baseline scenario</td>
<td>0.132</td>
<td>0.949</td>
<td>1.081</td>
<td>0.0024</td>
<td>1.11</td>
<td></td>
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<tr>
<td>Explosive path</td>
<td>3.924</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>0.89</td>
<td>3.843</td>
<td>Inf</td>
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<tr>
<td>Higher $\theta_\pi$</td>
<td>8.060</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>0.65</td>
<td>8.036</td>
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<td>Lower $\theta_\pi$</td>
<td>1.175</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>1.07</td>
<td>1.051</td>
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</table>
## Balance Sheet Implications /Alternative Money Demand

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(6)</th>
<th>(7)</th>
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<tr>
<td></td>
<td>$qB/P$</td>
<td>$-V/P$</td>
<td>$\text{PDV}$</td>
<td>$\text{seig.}$</td>
<td>$(1)+(2)$</td>
<td>$\overline{\tau}$</td>
<td>$C$</td>
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<td>0.132</td>
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<td>1.081</td>
<td>0.0024</td>
<td>1.11</td>
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<tr>
<td>Inflation scare</td>
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<td>0.951</td>
<td>0.993</td>
<td>0.0022</td>
<td>0.94</td>
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<td>Inf</td>
<td>Inf</td>
<td>0.89</td>
<td>3.843</td>
<td>Inf</td>
</tr>
</tbody>
</table>

Money demand = 0 for $r > 100\%$

<table>
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<tr>
<th></th>
<th>(10)</th>
<th>(11)</th>
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<tbody>
<tr>
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<td>0.396</td>
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<td>-0.041</td>
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A scatter plot of short term interest rates and M/PC
/Alternative Money Demand

Model
pre 1980 data
post 1980 data

Del Negro, Sims
Central bank’s balance sheet
Conclusions

Four levels of CB balance sheet problems: How likely?

1 Level 1: Accounting capital (book value of $B^C - V - M$) $< 0$. Or rule for remittances implies $\tau^C = 0$ for some period.
   - Likely (as in Carpenter et al.), but do not require fiscal support. In fact, PDV of remittances may actually increase.

2 Level 2: $qB^C - V < 0$
   - Possible, but not likely. In any case, no fiscal support required as long as PDV seignorage $\int_0^{\infty} \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s ds} dt > V - qB^C$.
   - The likelihood likely depends on conventional monetary policy ($\theta_\pi$).
Level 3: \[ qB^C - V + \int_0^\infty \frac{\dot{M}_t}{P_t} e^{-\int_0^t \rho_s \, ds} \, dt < 0 \]
- Unlikely – but very dependent on currency demand at high interest rates (which we do not know much about).

Level 4: Explosive paths/hyperinflation
- May not be ruled out without fiscal support / switch to active fiscal policy.