Managing Credit Bubbles

Alberto Martin       Jaume Ventura
CREI, UPF, and IMF   CREI and UPF

May 2014
Date Source - *Long series on credit to private non-financial sectors* by the Bank of International Settlements (2013). The plot above includes all OECD countries except Chile, Estonia, Israel, Slovak Republic and Slovenia. Slovak Republic joined the OECD in 2000, the rest in 2010.

Figure 1: Credit/GDP for OECD Countries
Figure 2: Selected OECD Countries with Credit/GDP trends

Date Source - Long series on credit to private non-financial sectors by the Bank of International Settlements (2013).
Introduction

- Fluctuations in credit are common (and more so in recent years)
  - Claessens et al. (2011): 114 credit cycles in 21 advanced economies (1960-2007)
  - Mendoza and Terrones (2012): 60 credit booms in 61 countries (1960-2010)
  - Dell’Ariccia et al. (2012): 175 credit booms in 170 countries (1960-2010)

- Closely linked to changes in real economic activity
  - asset prices higher during credit booms
  - real GDP and consumption growth higher during credit booms
  - investment growth twice as high during credit booms

- Source of concern:
  - credit booms end in crises and low growth (Schularick and Taylor, 2012)
  - role for policy? constrain boom to avoid bust
This paper

- Build on widespread view: credit largely driven by changes in borrowing constraints
  - “financial accelerator” literature

- Credit markets: borrowers exchange goods today for promises to deliver goods tomorrow
  - what backs these promises?
  - collateral, i.e. “pledgeable income” of borrowers
  - fluctuations in collateral key to understanding fluctuations in credit

- Our contribution:
  - distinguish between “fundamental” and “bubbly” collateral
    * fundamental collateral: credit backed by future output
      - fluctuations driven by changes in productivity, enforcement institutions, etc...
    * bubbly collateral: credit backed by expectation of future credit, i.e. by pyramid schemes
      - fluctuations driven by changes in expectations/investor sentiment

- questions
  * what determines an economy’s stock of bubbly collateral? when is it optimal?
  * what are the effects of bubbly collateral on investment, output and welfare?
Main results

- Study economy with weak enforcement institutions
  - insufficient fundamental collateral

- Important role for bubbly collateral, but mixed macroeconomic effects
  - crowding in: excess credit today, backed by expectation of future credit
    * raises investment and output
  - crowding out: past excess credit gets repaid today, diverts resources away from investment
    * lowers investment and output
  - “optimal” bubble trades off these effects to maximize long-term welfare

- Nothing guarantees that bubbly collateral supplied by market is optimal
  - role for collateral stabilization policy: “leaning against the wind”
    - fiscal backstop? yes

- Optimal bubble depends on location and riskiness
  - deposit vs. loan bubble
  - risky vs. safe bubble
Related literature

- Rational bubbles
  - Samuelson (1958): fiat money as a bubble
  - Blanchard and Watson (1982): bubbles in partial equilibrium

- Bubbles and financial frictions
  - Woodford (1990), Azariadis and Smith (1993), Woodford and Santos (1997): existence
  - Ventura (2011): cost of capital

- Credit booms
  - Gourinchas et al. (2001), Claessens et al. (2011), Dell’Ariccia et al. (2012), Mendoza and Terrones (2012), Ranciere et al. (2008)
Plan

1. The bubbly economy
2. A lender of last resort
3. The fiscal backstop
4. Financial intermediaries
5. Concluding remarks
The bubbly economy

- Two-period OLG. Generations contain workers/lenders and entrepreneurs/borrowers that maximize:

\[ U^i_t = C^i_{t,t} + \beta \cdot E^i_t C^i_{t,t+1} \]

- we measure generational welfare as \( U_t = \sum_i U^i_t \)

- Workers/lenders supply one unit of labor when young, receive wage \( W_t \) and decide how much to save:

\[ C^i_{t,t} = W_t - L_t \]

\[ C^i_{t,t+1} = R_{t+1} \cdot L_t \]

- workers/lenders save by purchasing credit contracts
- credit contracts offer (possibly contingent) return \( R_{t+1} \) \( \Rightarrow \) we call \( E_t R_{t+1} \) the interest rate
- workers/lenders maximize utility subject to budget constraints

- Entrepreneurs/borrowers sell credit contracts to construct portfolios of capital and bubbles (like real-world firms?):

- three choices: credit, capital, and bubbles
The bubbly economy (II)

- Entrepreneurs/borrowers invest and produce
  - investment technology:
    * produce capital for time $t + 1$ by investing consumption goods at time $t$ (one-to-one)
    * capital fully depreciates in production
  - production technology:
    \[ F(K_t, N_t) = A_t \cdot K_t^\alpha \cdot (\gamma^t \cdot N_t)^{1-\alpha} \]
    * $\gamma \geq 1$: growth of labor productivity
    * $A_t \in \{A_L, A_H\}$, with $A_L < A_H$ and $Pr(A_{t+1} \neq A_t) = \eta < 0.5$
The bubbly economy (III)

- Entrepreneurs/borrowers initiate and trade bubbles:
  - intrinsically useless asset: only held for resale, does not promise any payments

- Let $B_t$ denote value of bubbles in period $t$
  - some bubbles purchased from previous generations of entrepreneurs/borrowers
  - some bubbles initiated by current generation of old entrepreneurs/borrowers
  - aggregate bubble evolves as follows
    \[
    B_{t+1} = R_{t+1}^B \cdot B_t + B_{t+1}^N
    \]
    where
    * $R_{t+1}^B$ is return to bubbles purchased from generation $t - 1$;
    * $B_{t+1}^N$ is the value of bubbles initiated by generation $t$, “bubble creation”

- Two assumptions:
  - bubble is independent of individual actions
  - $B_{t+1}^N$ is random and non-negative
The bubbly economy (IV)

- Entrepreneurs/borrowers sell credit contracts to workers/savers
- Credit contracts need to be collateralized
  - interest payments can be contingent
  - weak enforcement institutions limit amount of collateral
    * entrepreneurs/borrowers can hide a fraction $1 - \phi$ of profits
- Credit or collateral constraint
  $$R_{t+1} \cdot L_t \leq \phi \cdot [F(K_{t+1}, N_{t+1}) - W_{t+1} \cdot N_{t+1}] + B_{t+1}$$

- Young and old-age budget constraints of entrepreneurs/borrowers given by
  $$C^{i}_{t,t} = L_t - K_{t+1} - B_t$$
  $$C^{i}_{t,t+1} = F(K_{t+1}, N_{t+1}) - W_{t+1} \cdot N_{t+1} + B_{t+1} - R_{t+1} \cdot L_t$$

- Entrepreneurs/borrowers maximize utility subject to credit and budget constraints
- From now on: lowercase letters to denote variables in efficiency units of labor, e.g. $k_t = \gamma^{-1}_t \cdot K_t$
Markets and prices

- **Labor market**: old entrepreneurs/borrowers (henceforth, borrowers) demand labor from young workers/lenders (henceforth, lenders)
  
  \[ w_t = (1 - \alpha) \cdot A_t \cdot k_t^\alpha \]

- **Market for bubbles**: old borrowers sell bubbles to young borrowers
  
  \[ E_t R^B_{t+1} = E_t R_{t+1} \]

- **Credit market**: young lenders give credit to young borrowers
  
  - **Supply of credit by young lenders**
    
    \[ l_t \begin{cases} = w_t & \text{if } \beta \cdot E_t R_{t+1} > 1 \\ \in [0, w_t] & \text{if } \beta \cdot E_t R_{t+1} = 1 \end{cases} \]

  - **Demand for credit by young borrowers**
    
    \[ R_{t+1} = \begin{cases} \alpha \cdot A_{t+1} \cdot k_{t+1}^{\alpha - 1} & \text{if } E_t b_{t+1}^N \geq (1 - \phi) \cdot \alpha \cdot E_t A_{t+1} \cdot k_{t+1}^\alpha \\ \frac{\phi \cdot \alpha \cdot A_{t+1} \cdot k_{t+1}^\alpha + b_{t+1}}{\gamma^{-1} \cdot l_t} & \text{if } E_t b_{t+1}^N < (1 - \phi) \cdot \alpha \cdot E_t A_{t+1} \cdot k_{t+1}^\alpha \end{cases} \]
Equilibrium dynamics

- Collapse previous equations as follows:

\[

t_{t+1} = \begin{cases} 
\frac{1 - \alpha \cdot A_t \cdot k_t^\alpha - b_t}{\gamma} & \text{if } \beta \cdot E_t R_{t+1} > 1 \\
\in \left[0, \frac{1 - \alpha \cdot A_t \cdot k_t^\alpha - b_t}{\gamma}\right] & \text{if } \beta \cdot E_t R_{t+1} = 1
\end{cases}
\]

(Supply of funds)

\[
E_t R_{t+1} = \min \{\alpha \cdot E_t A_{t+1}, E_t \{\phi \cdot \alpha + n_{t+1}\} \cdot A_{t+1}\} \cdot k_{t+1}^{\alpha-1}
\]

(Demand of funds)

\[
b_{t+1} = \frac{E_t R_{t+1} + u_{t+1}}{\gamma} \cdot b_t + n_{t+1} \cdot A_{t+1} \cdot k_{t+1}^\alpha
\]

(Bubble dynamics)

where

- \(u_{t+1}\) is unexpected component of bubble returns: \(u_{t+1} \equiv R_{t+1}^B - E_t R_{t+1}^B\)

- \(n_{t+1}\) is value of new bubbles as a share of output: \(n_{t+1} \equiv \frac{b_{t+1}^N}{A_{t+1} \cdot k_{t+1}^\alpha}\)

- Equilibria:

  - propose stochastic process for bubble shocks \(\{u_t, n_t\}\), satisfying \(E_t u_{t+1} = 0, b_t \geq 0\) and \(n_t \geq 0\)

  - search for sequence of state variables \(\{k_t, b_t\}\) that satisfies dynamic system with \(k_t \geq 0, b_t \geq 0\)

  - bubbleless equilibrium with \(\{u_t, n_t\} = \{0, 0\}\) always exists

  - but there are others!
Figure 3: Demand and supply of funds for investments
Figure 4: Demand and supply of funds in the presence of shocks
Bubbly equilibria

- Borrowers wish to purchase bubbles $\Leftrightarrow$ they grow as fast as the interest rate
- Borrowers can afford to purchase bubbles $\Leftrightarrow$ they do not grow faster than the economy
- Thus, bubbles can only exist if the rate of interest is lower than the growth rate of the economy
  - bubbles raise the interest rate $\Rightarrow$ condition must hold in bubbleless equilibrium
- Two possibilities:
  - interest rate is low because there is too much investment
  - interest rate is low because financial frictions limit fundamental collateral
    * focus on this last possibility, assuming
    $$\frac{1}{2} < \alpha < \frac{1}{1 + \phi}$$
Bubbly equilibria

- *Example 1*: economy with quiet bubble, $\{u_t, n_t\} = \{0, n\}$ for all $t$
  
  - constant productivity, $A_t = A$ for all $t$

- Main insight: there is an “optimal” size of the bubble
Figure 5: Deterministic Steady States
Figure 5: Deterministic Steady States
Bubbly equilibria (II)

• Example 2: economy with bubbly episodes:
  – constant productivity but different types of bubbles
  – $\varepsilon$ probability that a bubble pops up
  – $\delta$ is the probability of the bubble bursting
  – types of bubble:

<table>
<thead>
<tr>
<th>Size</th>
<th>Risk</th>
<th>safe</th>
<th>risky</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td></td>
<td>$(n_S, \delta_S)$</td>
<td>$(n_S, \delta_R)$</td>
</tr>
<tr>
<td>large</td>
<td></td>
<td>$(n_L, \delta_S)$</td>
<td>$(n_L, \delta_R)$</td>
</tr>
</tbody>
</table>

– simulate 10,000 periods of steady state behavior

• Main insights:
  – bubbly episodes give rise to macroeconomic fluctuations
  – the distance between equilibrium and “optimal” bubble varies over time in a complex way
Figure 6: Simulated economy with bubble shocks and constant productivity.
Figure 6: Simulated economy with bubble shocks and constant productivity
Bubbly equilibria (III)

- Example 3: economy with bubbly episodes and productivity shocks
- Main insights:
  - a bubble that is “too small” when $A_t = A_H$ might be “too large” when $A_t = A_L$
  - relative strength of crowding-in and crowding-out effects depends on fundamentals
- The bubble that attains full intermediation:
  - grows during booms and shrinks during recessions
  - accentuates effects of productivity shocks
Figure 7: Simulated economy with bubble and productivity shocks
Figure 7: Simulated economy with bubble and productivity shocks

<table>
<thead>
<tr>
<th></th>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{t+1}$</td>
<td>$\mu$ = 0.809</td>
</tr>
<tr>
<td></td>
<td>$s.d.$ = 0.288</td>
</tr>
<tr>
<td>$E_tU_{t+1}$</td>
<td>$\mu$ = 6.396</td>
</tr>
<tr>
<td></td>
<td>$s.d.$ = 1.618</td>
</tr>
</tbody>
</table>
Discussion

• Where is the market for bubbles?

• Slight variation of the model
  – assume production organized in firms, created and traded by borrowers
  – firms combine capital (partial depreciation) and bubbles

• In this variation, bubbles reflected in credit market:
  – loans to firms in excess of the NPV of future profits because lenders expect firms to obtain ‘excessive’
    loans also in the future
A lender of last resort

- Bubbly economy: shortage of fundamental collateral
  - bubbles can help, but they can also hurt capital accumulation and growth

- Can policy provide the optimal amount of collateral?
  - introduce agency to manage collateral through credit market interventions

- Lender of last resort (LOLR) that
  - taxes borrowing by young borrowers / subsidizes repayment by old borrowers
  - no direct effect on bubble, but allow for management of collateral
Bubbly economy with LOLR

- Let $S_t$ denote transfers to old borrowers, financed by taxes $X_t$ on young borrowers:
  - $S_t$ possibly contingent, i.e. guarantees
  - no bailouts, i.e. no net transfers to borrowers from other agents
  - balanced budget: $S_t = X_t$

- No direct impact on lenders

- Wealth of borrowers affected: policy provides resources

$$S_{t+1}^N = S_{t+1} - E_t R_{t+1} \cdot X_t$$

to borrowers of generation $t$, “collateral creation” by LOLR

- Borrower credit constraint becomes:

$$R_{t+1} \cdot L_t \leq \phi \cdot [F(K_{t+1}, N_{t+1}) - W_{t+1} \cdot N_{t+1}] + B_{t+1} + S_{t+1}$$
Markets and prices with a LOLR

- **Labor market**: as before

- **Market for bubbles**: as before

- **Credit market**:
  - supply of credit by young lenders: as before
  
  - demand for credit by young borrowers affected by intervention

\[
R_{t+1} = \begin{cases} 
\alpha \cdot A_{t+1} \cdot k_{t+1}^{\alpha-1} & \text{if } E_t (b_{t+1}^N + s_{t+1}^N) \geq (1 - \phi) \cdot \alpha \cdot E_t A_{t+1} \cdot k_{t+1}^{\alpha} \\
\phi \cdot \alpha \cdot A_{t+1} \cdot k_{t+1}^{\alpha} + b_{t+1} + s_{t+1} \over k_{t+1} + \gamma^{-1} \cdot (b_t + s_t) & \text{if } E_t (b_{t+1}^N + s_{t+1}^N) < (1 - \phi) \cdot \alpha \cdot E_t A_{t+1} \cdot k_{t+1}^{\alpha} 
\end{cases}
\]
Dynamics and welfare with a LOLR

- Collapse previous equations as follows:

\[
{k}_{t+1} = \begin{cases} 
(1 - \alpha) \cdot A_t \cdot k_t^\alpha - b_t - s_t & \text{if } \beta \cdot E_t R_{t+1} > 1 \\
0, \frac{(1 - \alpha) \cdot A_t \cdot k_t^\alpha - b_t - s_t}{\gamma} & \text{if } \beta \cdot E_t R_{t+1} = 1
\end{cases}
\]

(Supply of funds)

\[
E_t R_{t+1} = \min \{\alpha \cdot E_t A_{t+1}, E_t \{(\phi \cdot \alpha + n_{t+1} + m_{t+1}) \cdot A_{t+1}\}\} \cdot k_{t+1}^{\alpha-1}
\]

(Demand of funds)

\[
b_{t+1} = \frac{E_t R_{t+1} + u_{t+1}}{\gamma} \cdot b_t + n_{t+1} \cdot A_{t+1} \cdot k_{t+1}^\alpha
\]

(Bubble dynamics)

\[
s_{t+1} = \frac{E_t R_{t+1}}{\gamma} \cdot s_t + m_{t+1} \cdot A_{t+1} \cdot k_{t+1}^\alpha
\]

(Policy dynamics)

where

- \(m_{t+1}\) is the policy instrument, value of \(s^N_{t+1}\) as a share of output: \(m_{t+1} \equiv \frac{s^N_{t+1}}{A_{t+1} \cdot k_{t+1}^\alpha}\)

- Dynamic effects of LOLR policy mimic those of bubble shocks
  - past policy choices embedded in \(s_t\): reduce supply of funds for investment
  - future policy choices as captured in \(m_{t+1}\): raise collateral and demand for investment

- Construction of equilibria as before, given stochastic process \(\{m_t\}\) for policy
Policy

- LOLR can replicate any equilibrium of original economy
  - consider economy characterized by process \{u_t, n_t\} and corresponding bubble \(b_t\)
  - it is possible to replicate equilibrium under alternative process \{\hat{u}_t, \hat{n}_t\}, and corresponding bubble \(\hat{b}_t\), by setting \(m_t\)
    
    \[
    m_t \cdot A_t \cdot k_t^\alpha = \frac{\hat{u}_t \cdot \hat{b}_{t-1} - u_t \cdot b_{t-1}}{\gamma} + (\hat{n}_t - n_t) \cdot A_t \cdot k_t^\alpha
    \]

- If markets provide too little collateral or too much of it....consider policies to stabilize it!

- In particular, set \(\{m_t\}\) to satisfy

  \[
  E_t \{m_{t+1} \cdot A_{t+1}\} = \beta^{-1} \cdot k_{t+1}^{1-\alpha} - E_t \{(\phi \cdot \alpha + n_{t+1}) \cdot A_{t+1}\}
  \]

  \[
  k_{t+1} = \frac{(1 - \alpha) \cdot A_t \cdot k_t^\alpha - b_t - s_t}{\gamma}
  \]

  - stabilizes \(E_t R_{t+1} = \beta^{-1}\) at all times in all periods
  - guarantees full intermediation of wages

- "Leaning against the wind": policy rule
  - ‘complements’ bubble when collateral is scarce by subsidizing credit
  - ‘counteracts’ bubble when collateral is abundant by taxing credit
Bubbly equilibria with policy

- Example 1: economy with quiet bubble, \( \{u_t, n_t\} = \{0, n\} \) for all \( t \)

- Let \( n^* \) denote the optimal bubble in this example

- Effects of proposed policy rule \( \Rightarrow \) set \( m = n^* - n \) in all periods
  
  - policy raises steady state level of capital and welfare and sets \( \beta \cdot E_t R_{t+1} = 1 \)
  
  - if bubble is small, policy raises intermediation by setting \( m > 0 \)
  
  - if bubble is large, policy lowers interest rate by setting \( m < 0 \)
Figure 8: Effect of policy in an economy with bubble shocks and constant productivity
Bubbly equilibria with policy (II)

- **Example 2**: economy with bubbly episodes

- Same as before, policy rule sets

\[
m_t \cdot A_t \cdot k_t^\alpha = (n^* - n_t) \cdot A_t \cdot k_t^\alpha - \frac{u_t \cdot b_{t-1}}{\gamma}
\]

to replicate allocation that would arise under quiet bubble \{0, n^*\}

- This policy rule
  - sets \( \beta \cdot E_t R_{t+1} = 1 \) in all periods
  - stabilizes and raises steady state level of capital
  - stabilizes steady state consumption and raises average welfare
  - Note: not everyone is happy!
Figure 9: Simulated economy with bubble shocks and constant productivity (with and without policy)
Bubbly equilibria with policy (III)

- Economy with productivity shocks (A):
  - collateral is relatively scarce during booms
  - effects of rule are more complicated due to fundamental uncertainty

- Proposed policy rule
  - raises average capital stock and welfare
  - effect on volatility is ambiguous:
    * stabilizes effects of bubble shocks
    * amplifies effects of productivity shocks
Figure 10: Simulated economy with bubble and productivity shocks (with and without policy)
Fiscal backstop

- Consider LOLR has a taxation “capacity” $\tau$: $s_t < \tau$ in all periods
- Policy rule might require public debt
  - what changes? not much, but debt purchases voluntary
- Does lender demand for debt at $t$ depend on expected demand at $t + 1$?
  - is public debt a bubble?
  - if so, ability to intervene depends on sentiment (just like market credit)
Fiscal backstop

- Consider LOLR has a taxation “capacity” \( \tau \): \( s_t < \tau \) in all periods

- Policy rule might require debt: let \( D_t \) denote debt payments at \( t \)
  - LOLR budget constraint becomes
    \[
    S_t + D_t \leq X_t + q_t D_{t+1}
    \]
    where \( q_t \) is the price of a unit of public credit or debt

- Now, additional market for public credit: in equilibrium,
  \[
  q_t = \frac{1}{E_t R_{t+1}}
  \]

- Use \( D_{t+1}^N \) to denote
  \[
  D_{t+1}^N \equiv D_{t+1} - E_t R_{t+1} \cdot D_t,
  \]
  i.e., resources that debt policy transfers to generation \( t \)

  - \( D_{t+1}^N \) is the difference between the debt that generation \( t \) inherits and the debt it leaves behind
  - if \( D_{t+1}^N > 0 \), debt policy creates collateral in period \( t \)
  - if \( D_{t+1}^N < 0 \), debt policy destroys collateral in period \( t \)
Fiscal backstop (II)

- Law of motion of the system now given by:

\[
\begin{align*}
    k_{t+1} & \begin{cases} 
        = \frac{(1 - \alpha) \cdot A_t \cdot k_t^\alpha - b_t - s_t - d_t}{\gamma} & \text{if } \beta \cdot E_t R_{t+1} > 1 \\
        \in \left[0, \frac{(1 - \alpha) \cdot A_t \cdot k_t^\alpha - b_t - s_t - d_t}{\gamma}\right] & \text{if } \beta \cdot E_t R_{t+1} = 1
    \end{cases} \\
    E_t R_{t+1} &= \min \left\{ \alpha \cdot E_t A_{t+1}, E_t \left\{ \left( \phi \cdot \alpha + n_{t+1} + m_{t+1}^S + m_{t+1}^G \right) \cdot A_{t+1} \right\} \right\} \cdot k_{t+1}^{\alpha-1}
\end{align*}
\]

(Supply of funds)

(Demand of funds)

plus bubble and subsidy dynamics, where

- \( m_{t+1}^G \) is the new policy instrument, value of \( d_{t+1}^N \) as a share of output: \( m_{t+1}^G \equiv \frac{d_{t+1}^N}{A_{t+1} \cdot k_{t+1}^\alpha} \)

- Everything as before!

  - any subsidy policy \( s_t' \) can be replicated with subsidy and debt policy \( s_t + d_t = s_t' \)
  - fast-growing debt (i.e. \( d_{t+1}^N > 0 \)) makes it possible to provide subsidies with low taxes
  - but is debt prone to roll over crises, like bubbles?
Fiscal backstop (III)

- Is debt prone to roll-over crises?

- Consider equilibrium under optimal policy rule \((\beta \cdot E_t R_{t+1} = 1)\)

- Maximum payments that can be credibly promised at \(t + 1\) given by

\[
d_t \leq \beta \cdot \tau + \gamma \cdot \beta^2 \cdot E_t \{d_{t+1}\}
\]

so that, iterating forward,

\[
d_t \leq \tau \cdot \beta \cdot \sum_{n=0}^{\infty} (\gamma \cdot \beta)^n + \beta \cdot \lim_{s \to \infty} (\gamma \cdot \beta)^s E_t d_{t+s}.
\]

- But \(\gamma \cdot \beta > 1\): fiscal backstop unlimited!

  - same conditions that make bubbles possible imply fiscal backstop is unlimited
  - intuition: fiscal revenues grow at rate \(\gamma\), which is higher than the interest rate
    * no matter how small \(\tau\) is, NPV of taxation is infinite
    * unbounded backing for LOLR’s debt
Discussion

• How naive is this view?
  – LOLR may face credibility issues, reducing the extent to which it can pledge future revenues
  – LOLR may be inefficient in taxing/distributing resources
  – both factors limit its ability to replicate the optimal bubble

• But main insight remains
  – given limitations, choose desired/feasible level of intervention
  – desired intervention can always be financed by LOLR
Extensions

• Up to this point:
  – bubbly collateral matters to sustain credit
  – LOLR can improve on market outcomes through credit market interventions

• Sometimes not just size, but also type of bubble matters

• Two extensions: introduce
  – financial intermediaries: collateral needed to sustain both, loans and deposits
  – risk averse lenders: risky and safe collateral have different properties
  – message: it is not just total collateral that matters, but also its location and type
Extension 1: financial intermediaries

- Assume $A_t = 1$ and $\phi = 0$

- Introduce intermediaries: subset of individuals that can lend to borrowers
  - in markets for deposits, sell credit contracts to lenders at deposit rate $E_t R_{t+1}^D$
  - in markets for loans, purchase credit contracts from borrowers at lending rate $E_t R_{t+1}^L$

- Deposit ($B_t^D$) and loan ($B_t^L$) bubbles, traded by intermediaries and borrowers, evolve according to
  \[
  B_{t+1}^D = R_{t+1}^{BD} \cdot B_t^D + B_{t+1}^{ND}
  \]
  \[
  B_{t+1}^L = R_{t+1}^{BL} \cdot B_t^L + B_{t+1}^{NL}
  \]
  where $R_{t+1}^{BD}$ and $R_{t+1}^{BL}$ are returns to bubbles purchased from generation $t - 1$; and $B_{t+1}^{ND}$ and $B_{t+1}^{NL}$ reflect bubble creation.

- Both loan and deposit contracts must be collateralized, so that
  \[
  R_{t+1}^D \cdot D_t \leq B_{t+1}^D
  \]
  \[
  R_{t+1}^L \cdot L_t \leq B_{t+1}^L
  \]

- Equilibrium in markets for bubbles requires:
  \[
  E_t R_{t+1}^{BD} = E_t R_{t+1}^D
  \]
  \[
  E_t R_{t+1}^{BL} = E_t R_{t+1}^L
  \]
Extension 1: financial intermediaries

- Collapse previous equations as follows:

\[
\begin{align*}
  k_{t+1}^* & = \begin{cases} 
  \frac{(1 - \alpha) \cdot k_t^\alpha - b^D_t - b^L_t}{\gamma} & \text{if } \beta \cdot E_t R^D_{t+1} > 1 \\
  0, \frac{(1 - \alpha) \cdot k_t^\alpha - b^D_t - b^L_t}{\gamma} & \text{if } \beta \cdot E_t R^D_{t+1} = 1 
\end{cases} \\
  E_t R^D_{t+1} & = \min \left\{ E_t R^L_{t+1}, \frac{E_t n^D_{t+1} \cdot k_t^\alpha}{k_{t+1}^\gamma} \right\} \\
  E_t R^L_{t+1} & = \min \left\{ \alpha, E_t n^L_{t+1} \right\} \cdot k_t^{\alpha - 1} \\
  b^D_{t+1} & = \frac{E_t R^D_{t+1} + u^D_{t+1}}{\gamma} \cdot b_t^D + n^D_{t+1} \cdot k_t^\alpha \\
  b^L_{t+1} & = \frac{E_t R^L_{t+1} + u^L_{t+1}}{\gamma} \cdot b_t^L + n^L_{t+1} \cdot k_t^\alpha 
\end{align*}
\]

where \( \left\{ u^j_{t+1}, n^j_{t+1} \right\} \) are the bubble-return and creation shocks of bubbles of type \( j \in \{ L, D \} \)

- Main insight: not just amount, but also distribution of collateral matters

- Economy may find itself in region of partial intermediation because

  - intermediaries have insufficient collateral
    * deposit bubble creation \( (n^D_{t+1} > 0) \) is expansionary, loan bubble creation \( (n^L_{t+1} > 0) \) is contractionary
  
  - borrowers have insufficient collateral
    * loan bubble creation \( (n^L_{t+1} > 0) \) is expansionary
Extension 2: risk-averse lenders

- Assume $A_t = 1$ and $\phi = 0$

- Assume fraction $\rho$ of lenders is risk-averse, with preferences
  \[
  U_t^i = C_{t,t}^i + \beta \cdot \min_t C_{t,t+1}^i
  \]

- Borrowers sell two types of credit contracts, safe and risky, with constraints
  \[
  R_{t+1}^R \cdot L_t^R + R_{t+1}^S \cdot L_t^S \leq B_{t+1}
  \]
  \[
  R_{t+1}^S \cdot L_t^S \leq \min_t B_{t+1}
  \]

- Assume high ratio of risky/safe collateral
  - wages of risk-neutral lenders fully intermediated
  - wages of risk-averse lenders only partially intermediated
Extension 2: risk-averse lenders

- Model collapses to:

\[ k_{t+1} = \beta \cdot \min_t b_{t+1} + (1 - \rho) \cdot \frac{1 - \alpha}{\gamma} \cdot k_t^\alpha - \frac{b_t}{\gamma} \]

\[ E_t R_{t+1}^R = \frac{E_t n_{t+1} \cdot k_{t+1}^\alpha - \min_t b_{t+1}}{k_{t+1} - \beta \cdot \min_t b_{t+1}} \]

\[ b_{t+1} = \frac{E_t R_{t+1}^R + u_{t+1}}{\gamma} \cdot b_t + n_{t+1} \cdot k_{t+1}^\alpha \]

- Main insight:

  - it is not total stock of bubbly collateral that matters, only safe fraction
  - increase in safe collateral (\( \min_t n_{t+1} \)): expansionary
  - increase in risky collateral (\( E_t n_{t+1} - \min_t n_{t+1} \)): contractionary
  - policy implications
Concluding remarks

• Credit booms and busts are a fact of life in modern economies

• Widespread view among macroeconomists: fluctuations in collateral are an important part of story

• We build on this view to derive the following results:
  – economies with binding borrowing constraints: fundamental and bubbly collateral
    * both types of collateral drive credit
    * bubbly collateral driven by sentiments or expectations
  – bubbly collateral raises credit (“crowding-in”) but diverts part of it away from investment (“crowding-out”)
    * “optimal” bubble size trades off these two effects to maximize long-term output and welfare
  – markets are generically unable to provide the optimal amount of bubbly collateral
    * LOLR can replicate “optimal” bubble allocation through credit market interventions

• Limitations:
  – perfect information: how do we know whether fluctuations are driven by fundamental or bubbly collateral?
  – exogeneity of fundamental collateral: in reality, collateral is “produced” by the financial system through screening and monitoring of borrowers