Take the Short Route

How to repay and restructure sovereign debt with multiple maturities

Mark Aguiar Manuel Amador

Maturity Choice of Government Debt

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Sharp answer

- During deleveraging
  - Do not issue or buy back long-term bonds – only use short term bonds
- Leave any active maturity management to the end
- Market outcome is Pareto inefficient

Context of Environment

Legacy Debt

- "Ex Ante" tranquil regime
- Debt raised when threat of default was low
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Crisis Regime

- Now face a significant risk of default – primary concern
- What to do about inherited legacy debt?
- Goal is to isolate and analyze the equilibrium response to this threat

Basic Environment

- Discrete time, infinite horizon, small open economy
- Government preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$
- Risk-neutral lenders with opportunity cost $R = 1 + r = \beta^{-1}$
- Non-contingent bonds of different maturity
  - Benchmark: one-period (b_S) and perpetuity (b_L), with prices (q_S, q_L) and normalized coupons r
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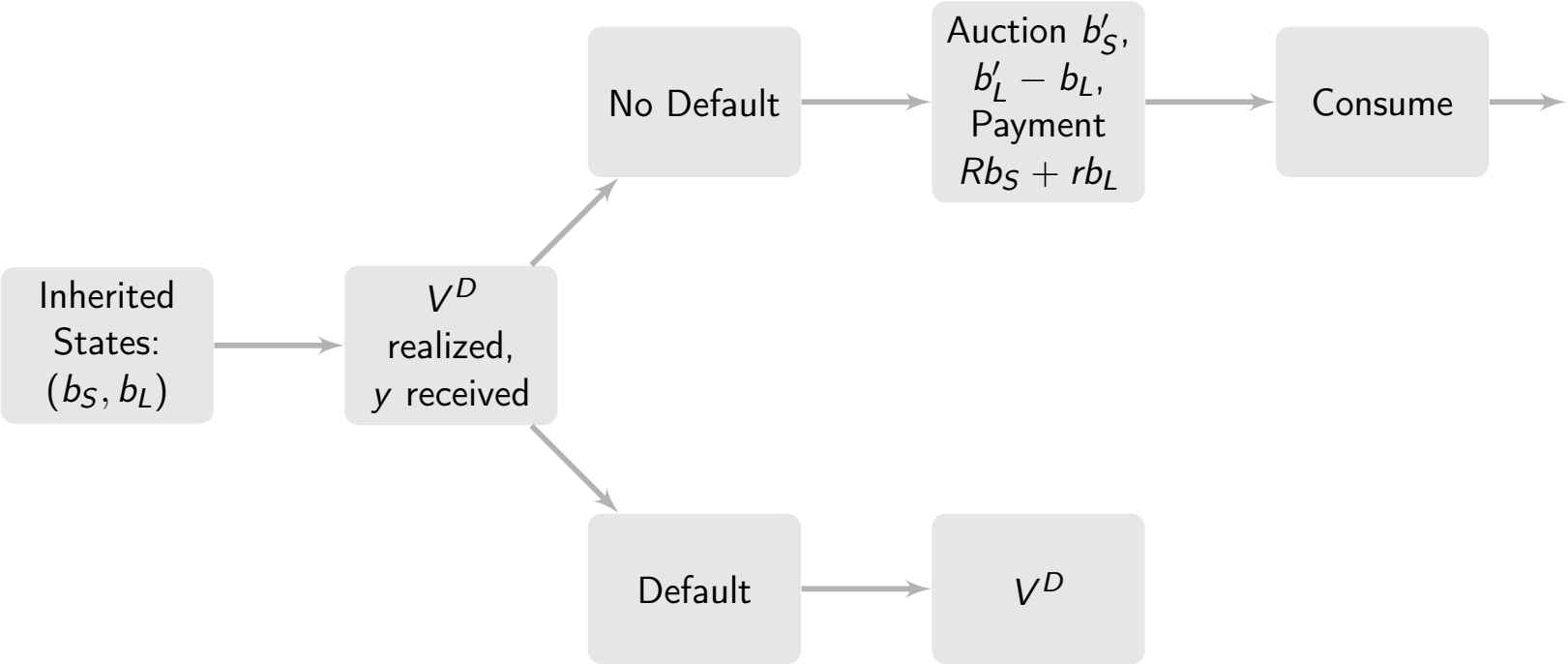
- Constant endowment $y \Leftarrow$ No Spanning Motive

Default

Limited commitment

- Country can default. Outside option: V^D
- V^D varies stochastically $\{\underline{V}^D, \overline{V}^D\}$. $\underline{V}^D < \overline{V}^D$.
- $\underline{V}^D$: strong enforcement regime – initial state
- Transition probability λ
- $\overline{V}^D$: weak enforcement regime – absorbing state (wlog)

Timing in the benchmark model



Towards a Markov Equilibrium

State

- $\boldsymbol{b} = (b_S, b_L)$, $\mathcal{B} = \mathbb{R}_+ \times \mathbb{R}_+$

Government's problem if repay today:

$$V(\boldsymbol{b}, V^D) = \max_{\{c \geq 0, \boldsymbol{b} \in \mathcal{B}\}} \left\{ u(c) + \beta \mathbb{E} \left[\max\langle V(\boldsymbol{b}', V^{D\prime}), V^{D\prime} \rangle \middle| V^D \right] \right\}$$

subject to:

$$c + (1+r)b_S + rb_L \leq y + q_S(V^D, \boldsymbol{b}')b_S' + q_L(V^D, \boldsymbol{b}')(b_L' - b_L).$$

Default:

- $D(\boldsymbol{b}, V^D) = 1$ if $V^D > V(\boldsymbol{b}, V^D)$, and zero otherwise

Towards a Markov Equilibrium II

Investors' break even conditions

$$q_S(V^D, \boldsymbol{b}') = \mathbb{E}\left[1 - D(\boldsymbol{b}', V^{D\prime}) \middle| V^D\right]$$

$$q_L(V^D, \boldsymbol{b}') = \mathbb{E}\left[\left(1 - D(\boldsymbol{b}', V^{D\prime})\right)\left(\frac{r + q_L'}{1 + r}\right) \middle| V^D\right]$$

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Time Consistency

- Long-term bonds present a time consistency issue – price today depends on future debt policies
- Short-term bonds are not subject to this issue – at time of issuance, default risk over the maturity of the bond fully pinned down independent of future fiscal policies

Markov Equilibrium

Definition 1

A Markov Perfect Equilibrium consists of policy functions C, B_S, B_L, and D, and pricing schedules q_S and q_L, such that for all debt positions $\boldsymbol{b} \in \mathcal{B}$ and $V^D \in \{\underline{V}^D, \overline{V}^D\}$, and absent a prior default:

(i) *the policy functions C, B_S, and B_L, solve the government's problem conditional on q_S and q_L;*

(ii) *D is an indicator function that takes one if $V(\boldsymbol{b}, V^D) < V^D$ and zero otherwise; and*

(iii) *the creditors' break-even conditions are satisfied with $q_i \in [0, 1], i = S, L$, given the government's policy functions.*

Regions

- No-Default

$$\boldsymbol{ND} = \{\boldsymbol{b} \in \mathcal{B} \big| V(\boldsymbol{b}, V^D) \geq V^D, \forall V^D\}.$$

  - Absorbing state with risk-free prices: $V = \frac{u(y - r(b_{\boldsymbol{S}} + b_{\boldsymbol{L}}))}{1-\beta}$
  - Boundary given by $\overline{B}$: $\frac{u(y - r\overline{B})}{1-\beta} = \overline{V}^D$
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- Crisis:

$$\boldsymbol{C} = \left\{ \boldsymbol{b} \in \mathcal{B} \middle| V(\boldsymbol{b}, \underline{V}^D) \geq \underline{V}^D \quad \& \quad V(\boldsymbol{b}, \overline{V}^D) < \overline{V}^D \right\}.$$

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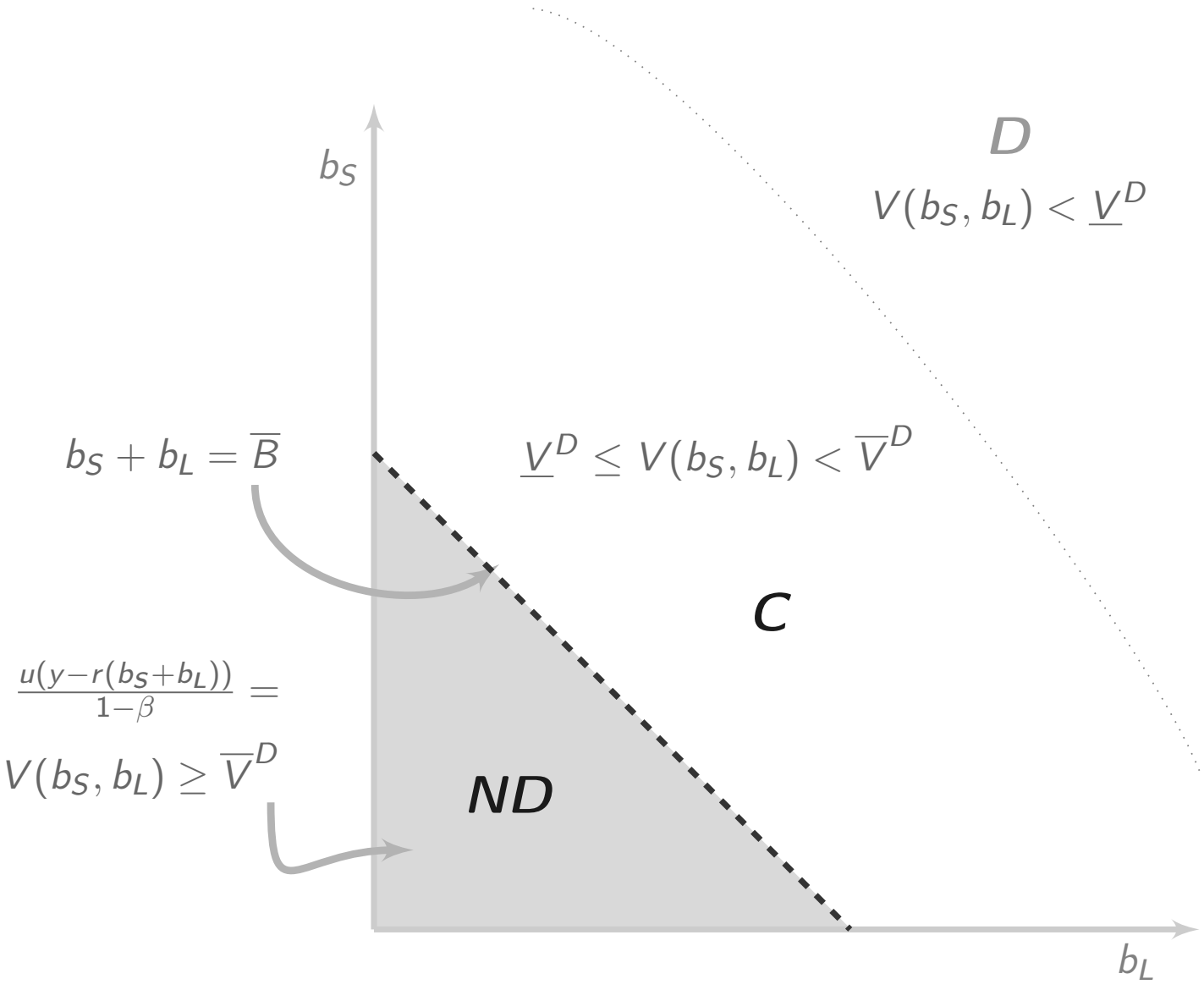
- Default: $\boldsymbol{D} = \{\boldsymbol{b} | V(\boldsymbol{b}, \underline{V}^D) < \underline{V}^D\}$.

Regions


b_S
D
$V(b_S, b_L) < \underline{V}^D$
$b_S + b_L = \overline{B}$
$\underline{V}^D \leq V(b_S, b_L) < \overline{V}^D$
C
$\frac{u(y-r(b_S+b_L))}{1-\beta} =$
$V(b_S, b_L) \geq \overline{V}^D$
ND
b_L

Markov Equilibrium Prices

Short-term Bonds

$$q_S(\boldsymbol{b}) = \mathbb{E}\left[1 - D(\boldsymbol{b}', V^{D\prime})\middle|\underline{V}^D\right] = \begin{cases} 1 & \text{if } \boldsymbol{b} \in \boldsymbol{ND} \\ 1-\lambda & \text{if } \boldsymbol{b} \in \boldsymbol{C} \\ 0 & \text{if } \boldsymbol{b} \in \boldsymbol{D} \end{cases}$$

Long-term Bonds

$$\begin{aligned} q_L(V^D, \boldsymbol{b}') &= \mathbb{E}\left[\left(1 - D(\boldsymbol{b}', V^{D\prime})\right)\left(\frac{r + q_L'}{1+r}\right)\middle|\underline{V}^D\right] \\ &= r\sum_{t=1}^{T(\boldsymbol{b})}\left(\frac{1-\lambda}{1+r}\right)^t + \left(\frac{1-\lambda}{1+r}\right)^{T(\boldsymbol{b})} \end{aligned}$$

- Where $T(\boldsymbol{b}) \in \{0, 1, ..., \infty\}$: equilibrium exit time from $\boldsymbol{C}$

Main Results

Result 1

- A weakly optimal equilibrium policy is to never issue or repurchase long-term bonds ($b'_L = b_L$)

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Result 2

- If $T(\boldsymbol{b}) < \infty$, this policy is strictly optimal

Outline of Proof for Result 1

- Pick an arbitrary (feasible) exit time T and substitute associated prices into budget set
- Any consumption stream that satisfies this budget set can be implemented using only short-term bonds (paying only the coupon on perpetuities)
  - As this is true for any feasible T, it holds for the equilibrium $T(\boldsymbol{b})$ as well, and thus the equilibrium consumption path can always be implemented trading only short-term bonds
- Equilibrium $T(\boldsymbol{b})$ is simply the max utility over all possible exit times implemented by trading short-term bonds

Intuition for Result 1

- Prices of short-term bonds are determined by outstanding debt at time of auction.
- No time-consistency issue – government can vary T without changing short-term bond prices conditional on debt.
- Short-term bond prices are actuarially fair for any T and thus the exit time that maximizes welfare is an (the) equilibrium.

The dummy maximization problem

- Suppose the government could commit to an exit-time T
- but restricted to use only short-term bonds

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$$W(\boldsymbol{b}, T) = \max_{\{b_{S,T}, \{c_t\}_{t=0}^{T-1}\}} \left\{ \sum_{t=0}^{T-1} \beta^t (1-\lambda)^t u(c_t) + \beta^T (1-\lambda)^{T-1} \times \right.$$
$$\left. \times \left(\frac{u(y - r(b_{S,T} + b_L))}{1-\beta} \right) + \sum_{t=1}^{T-1} \beta^t (1-\lambda)^{t-1} \lambda \overline{V}^D \right\}$$

subject to:

$$b_S \leq (1+r)^{-1} \left(\sum_{t=0}^{T-1} \left(\frac{1-\lambda}{1+r} \right)^t (y - c_t - r b_L) + \left(\frac{1-\lambda}{1+r} \right)^{T-1} b_{S,T} \right)$$

$$b_{S,T} \leq \overline{B} - b_L.$$

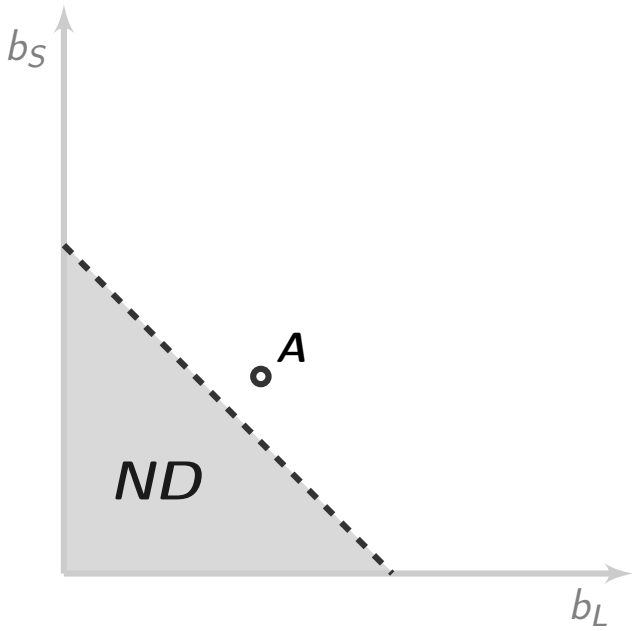
Main result #1: Two Lemmas

- Lemma 1: $V(\boldsymbol{b}, V^D) \leq W(\boldsymbol{b}, T(\boldsymbol{b}))$
- Lemma 2: $V(\boldsymbol{b}, V^D) \geq W(\boldsymbol{b}, T)$ for any T
- $\Rightarrow V(\boldsymbol{b}, V^D) = \sup_T W(\boldsymbol{b}, T)$

Alternative Maximization Problem

- Can solve maximization problem of paying down short-term bonds to exit in arbitrary period T: $W(\boldsymbol{b}, T)$
- $V(\boldsymbol{b}) = \sup_T W(\boldsymbol{b}, T)$
- Can solve government's problem without knowing long-term prices, and then with $T(\boldsymbol{b})$ we have q_L
- Avoids complicated fixed point problem on prices and policies
- Equilibrium is unique up to integer constraint on $T(\boldsymbol{b})$

Intermission – Why save?



- At point $\boldsymbol{A}$: $V(\boldsymbol{b}, \underline{V}^D) < \overline{V}^D$

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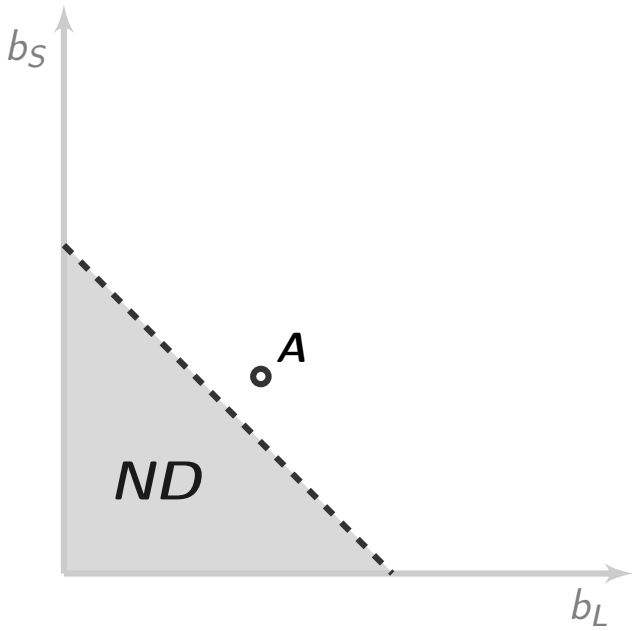

Stay put at A: $T = \infty$

$$c^{\infty} = y - r(b_S + b_L) - \lambda b_S$$

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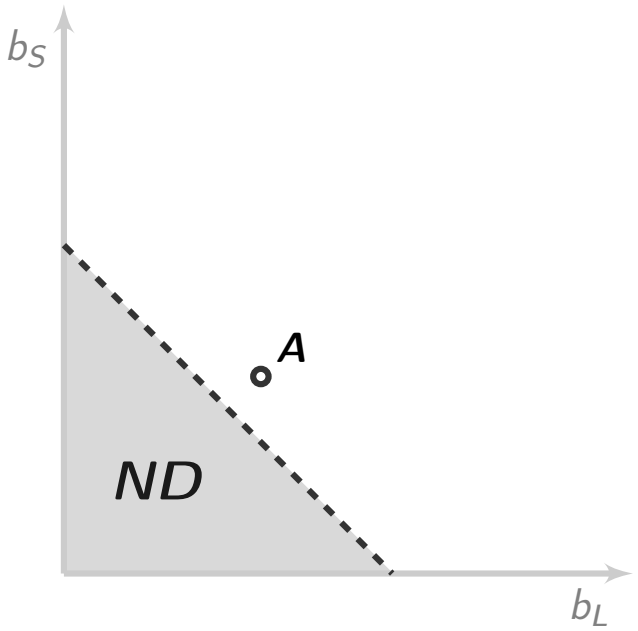

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Go to ND in one step: $T = 1$

$$c^{1} = y - r(b_S + b_L) - (b_S + b_L - \overline{B})$$

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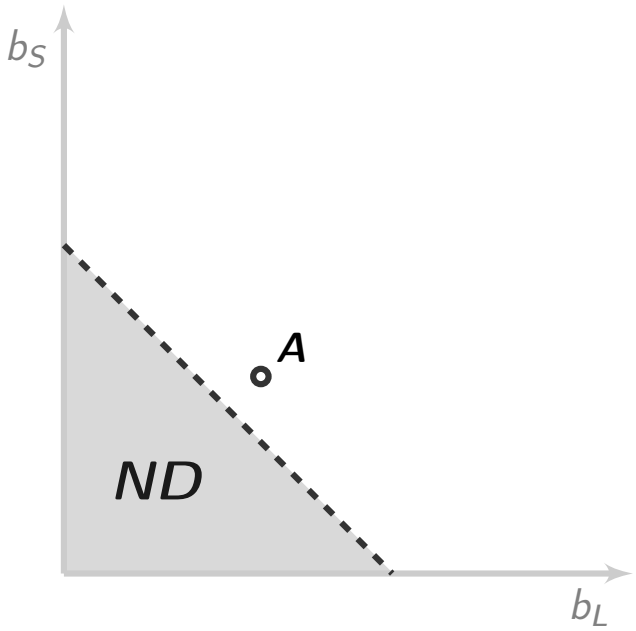

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- As $b_S + b_L \to \overline{B}$, strictly prefer to exit if $b_S >> 0$

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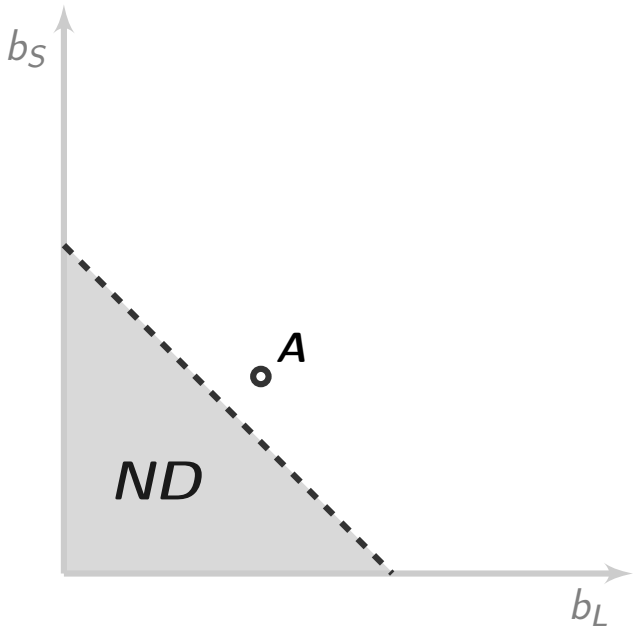

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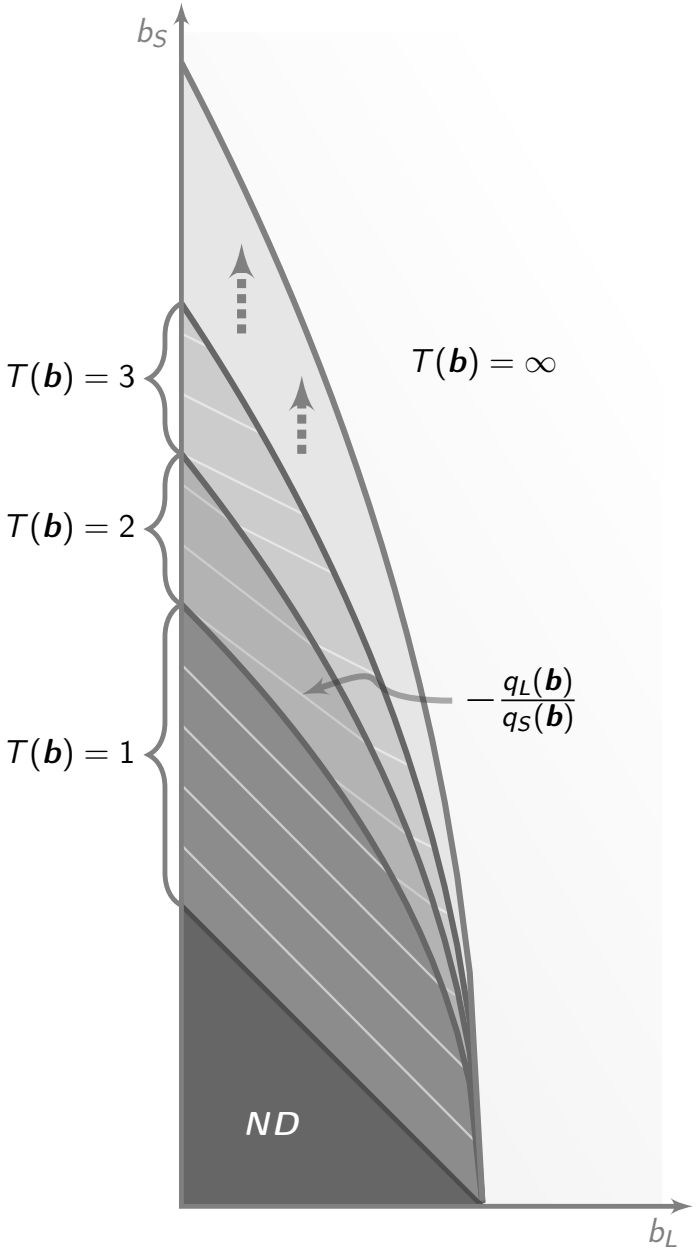
$$\lambda b_S \gtrless b_S + b_L - \overline{B}$$

- As $b_S + b_L \to \overline{B}$, strictly prefer to exit if $b_S >> 0$
- Default is not a zero-sum game
- Short-term debt puts the full cost of potential default on the sovereign

Equilibrium Iso-T regions



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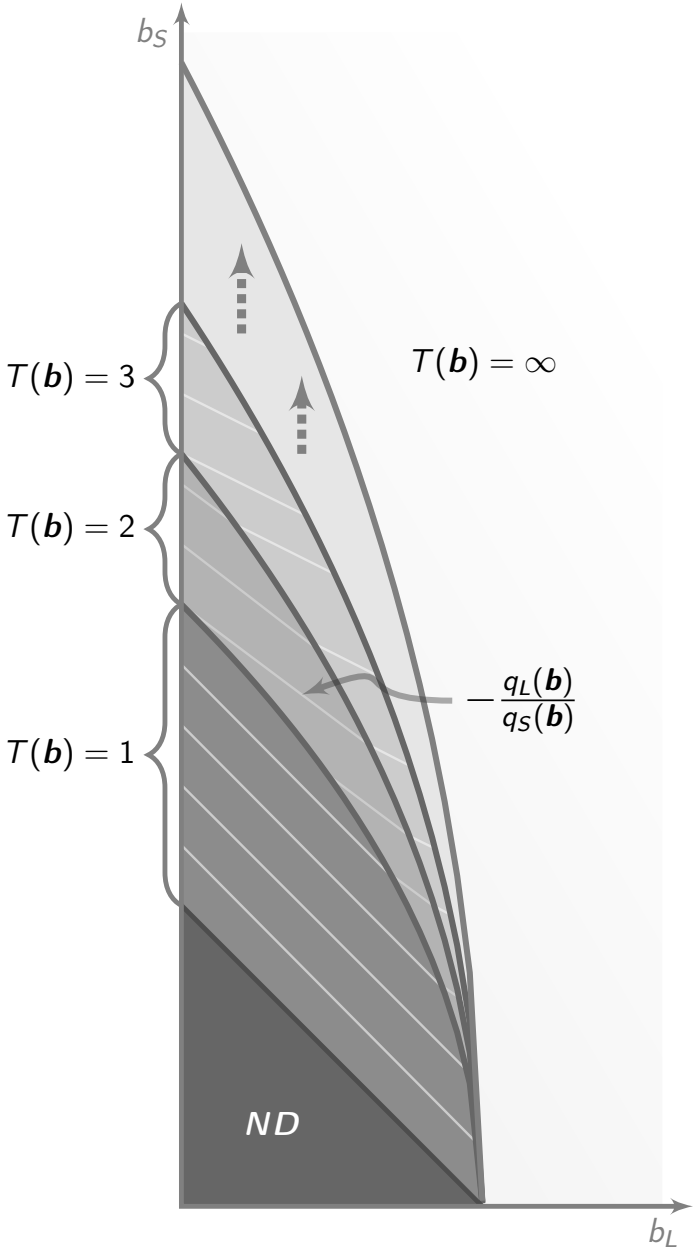

Speed of exit affected by maturity composition

More long term bonds: slower exit

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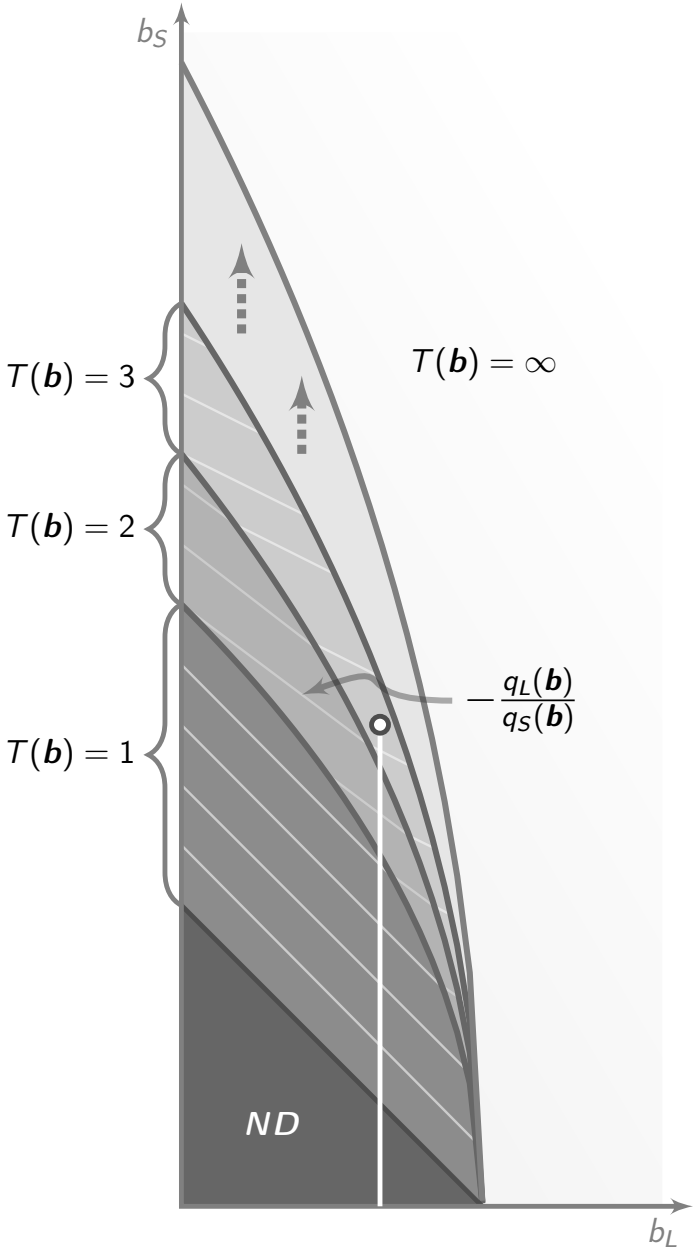

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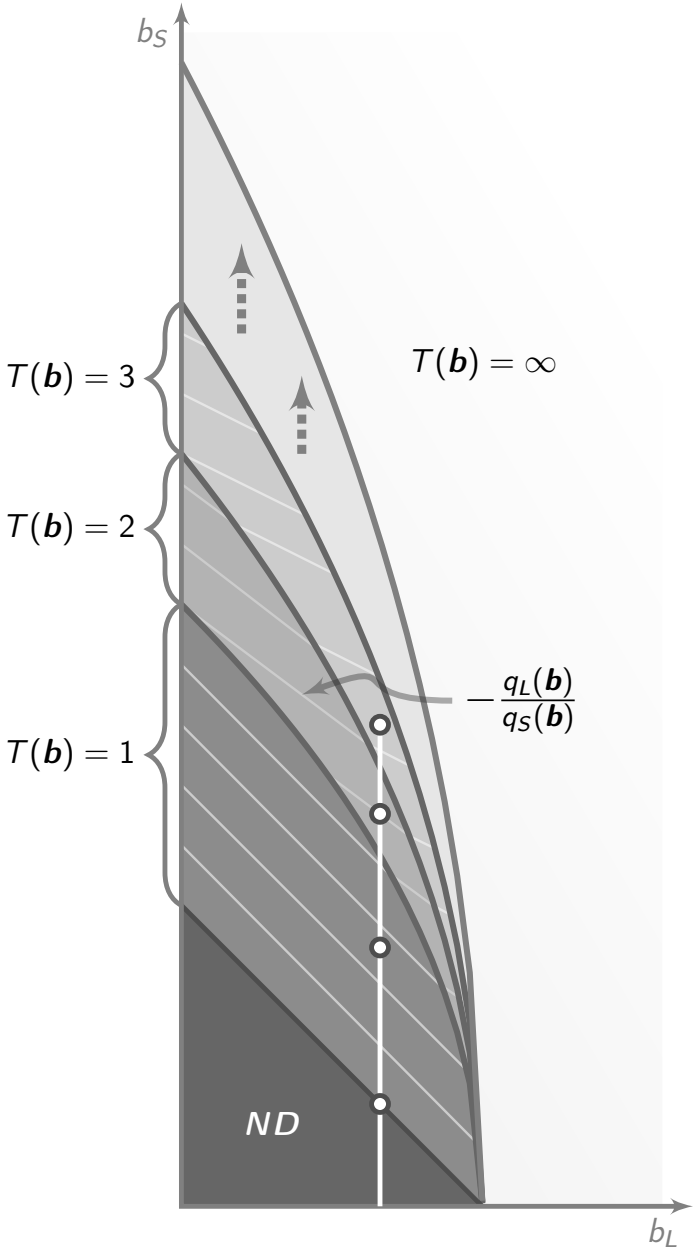

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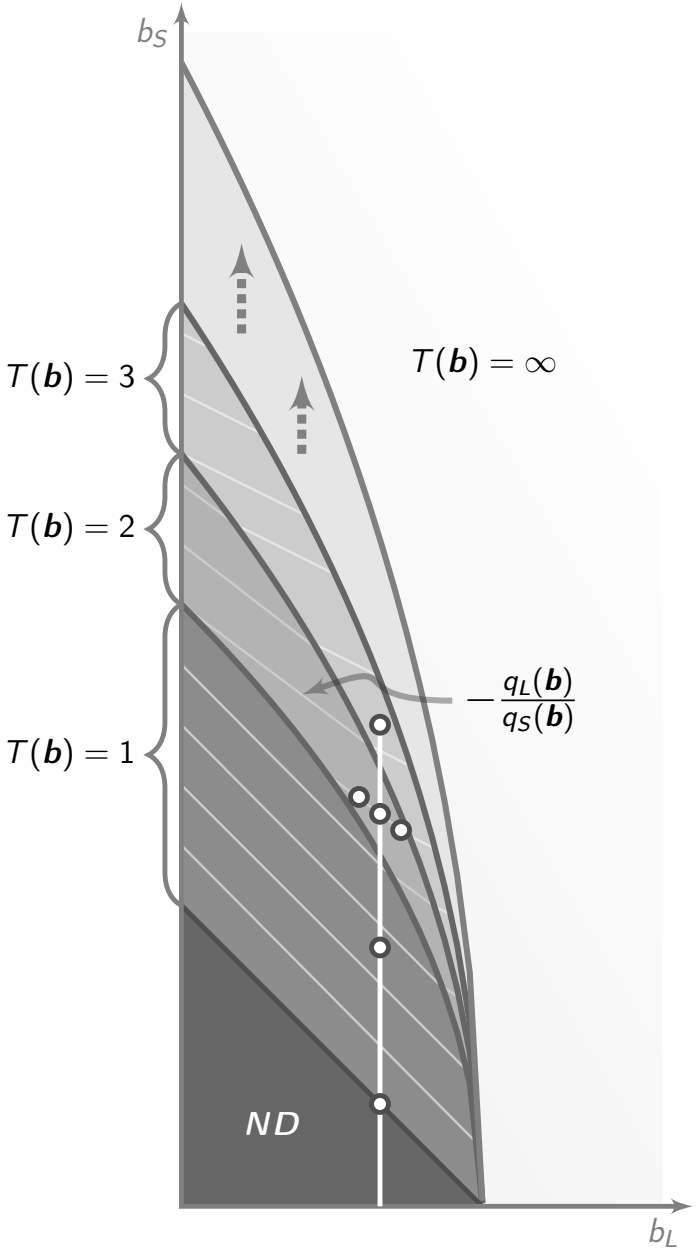

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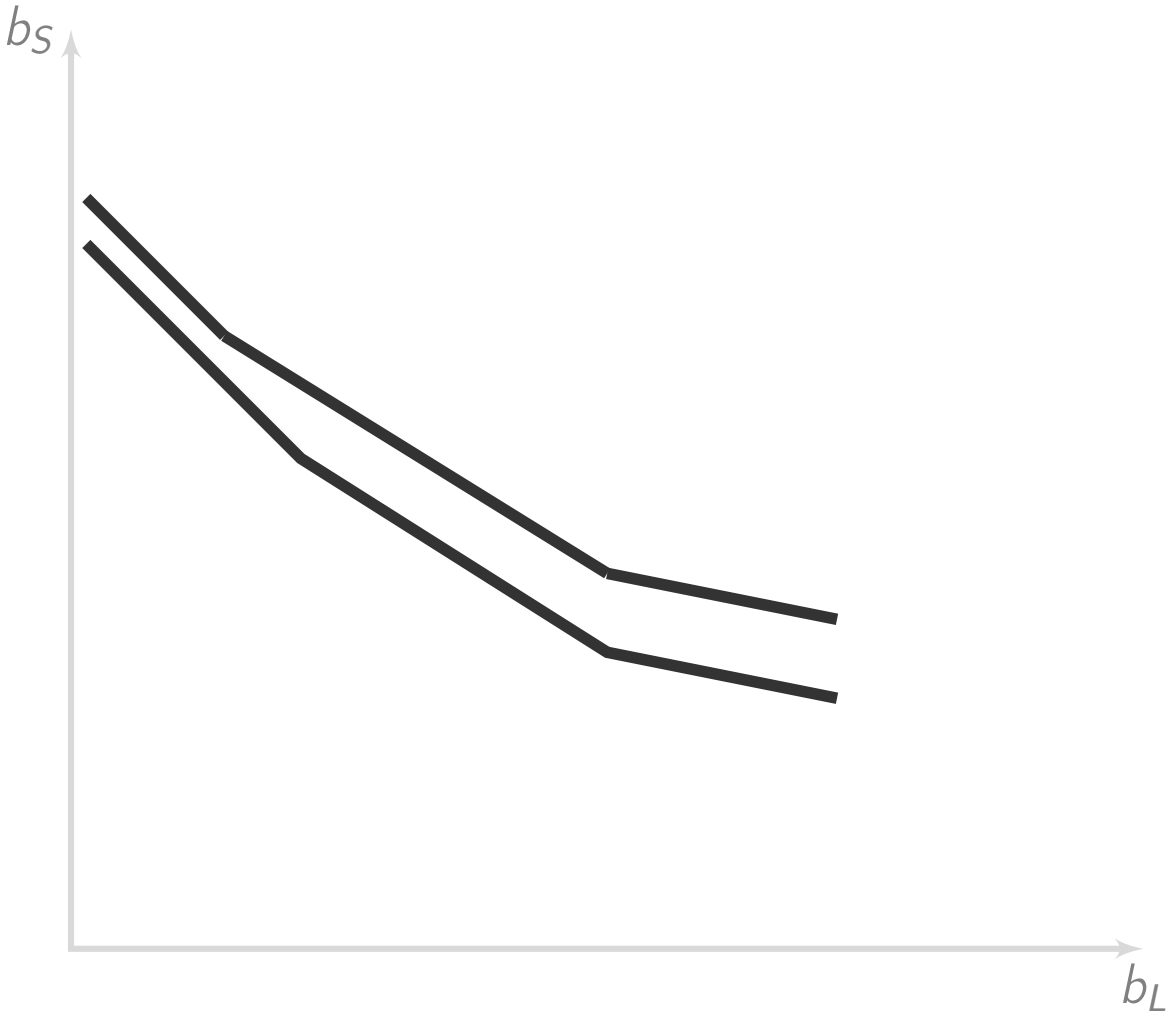
Link to Efficient Contracting

- Hopenhayn and Werning consider an optimal contract between a principal and agent in this framework (with unobservable outside option shocks)
- Show that it can be implemented using short-term bonds
- Did not consider legacy debt or multiple principals, but extension implies our result is more general than our incomplete contracting space –> the four of us are working on this now so stay tuned
- Raises the question about why not dilute (or buy back) previous bond holders?

Equilibrium Iso-V curves



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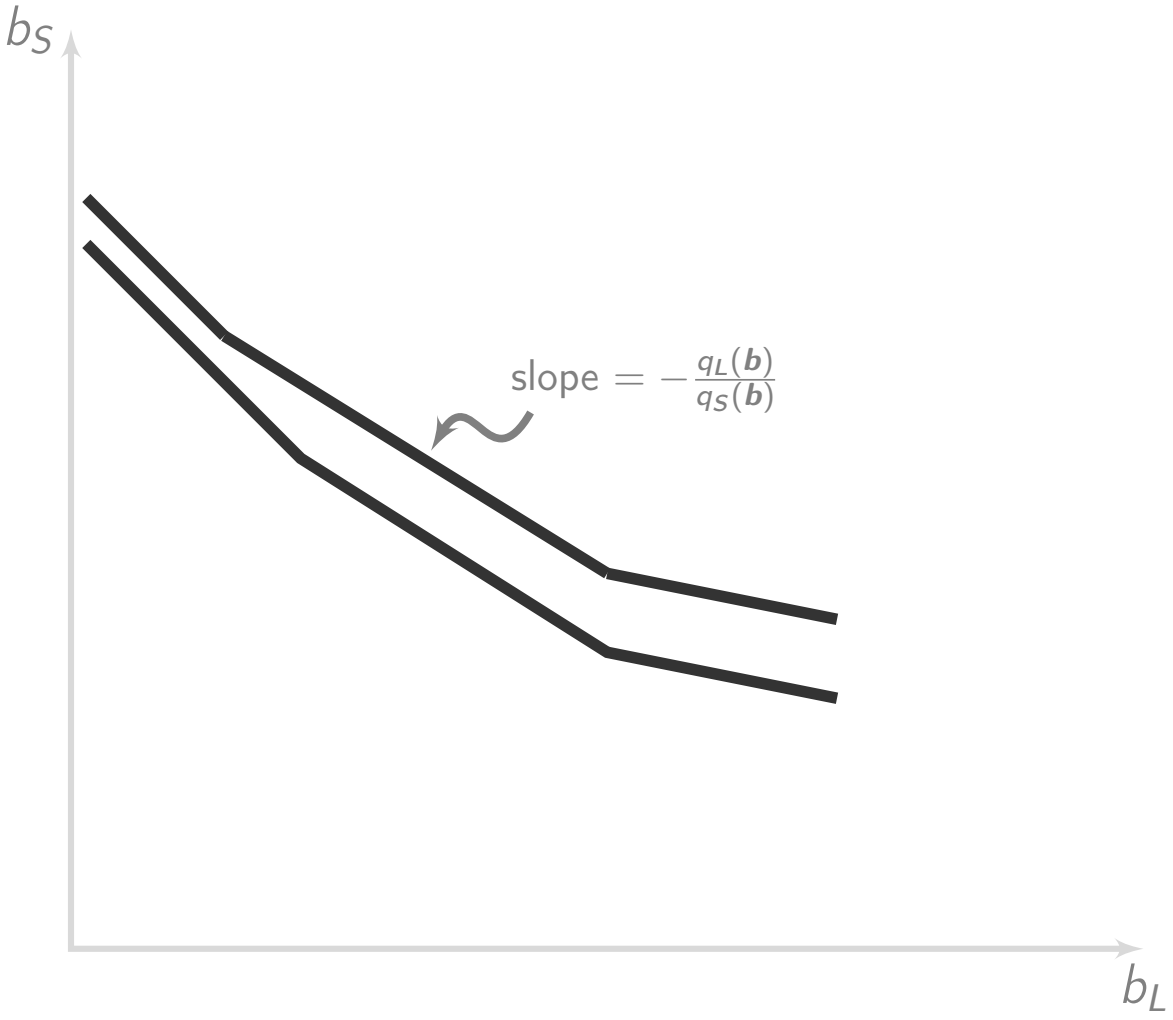

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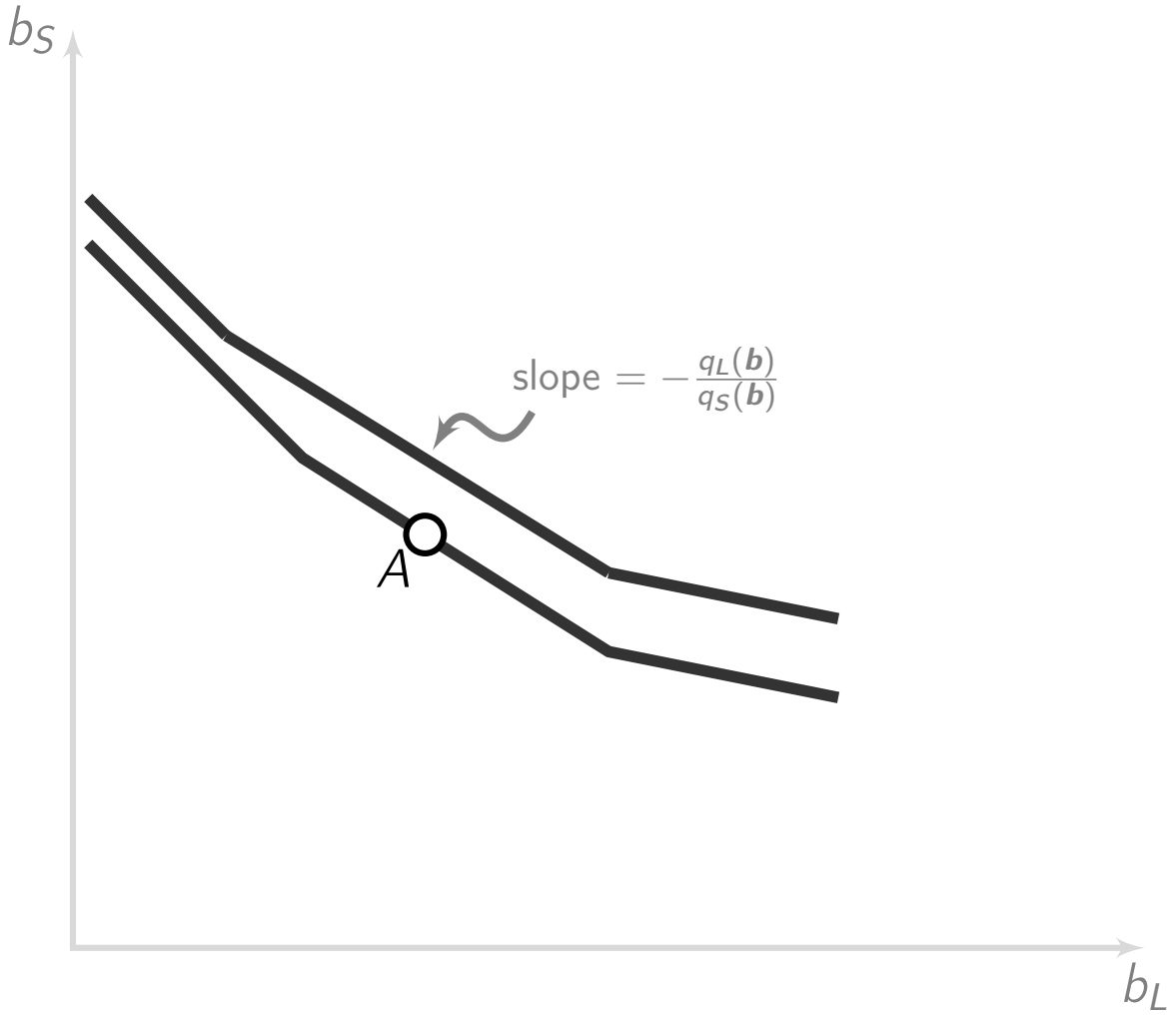

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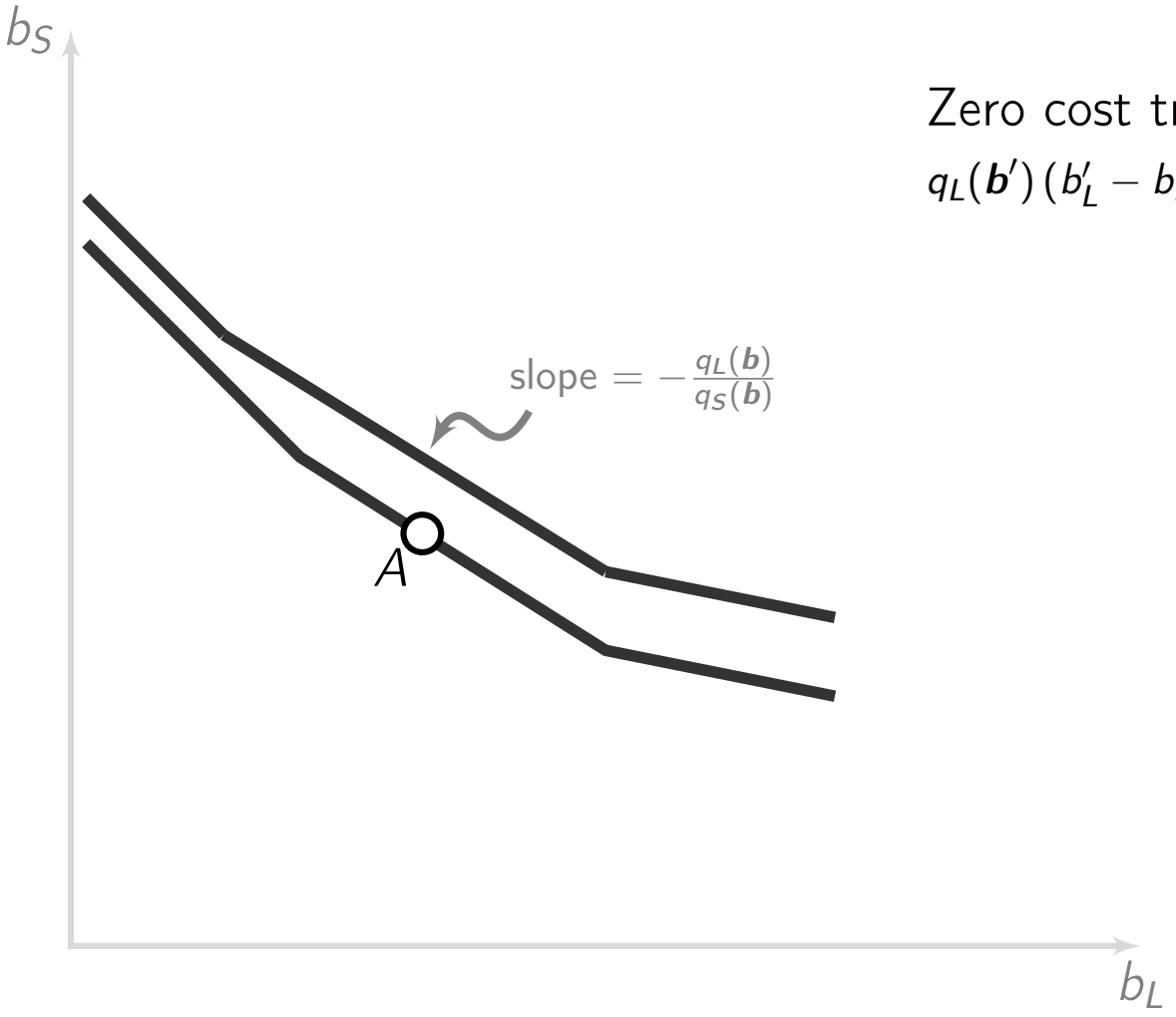

Zero cost trades

$q_L(\boldsymbol{b}')\,(b'_L - b_L) + q_S(\boldsymbol{b}')\,(b'_S - b_S) = 0$

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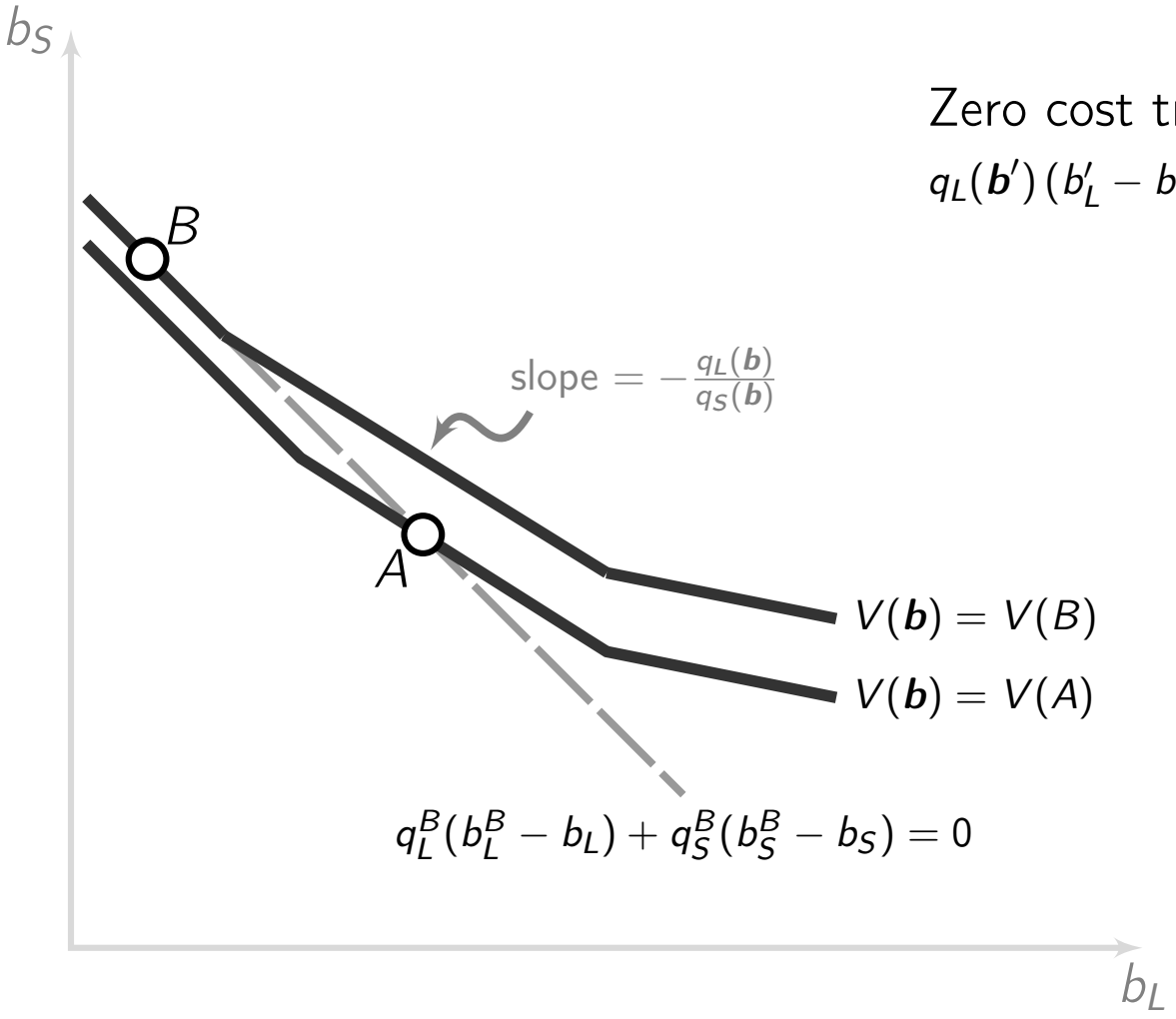

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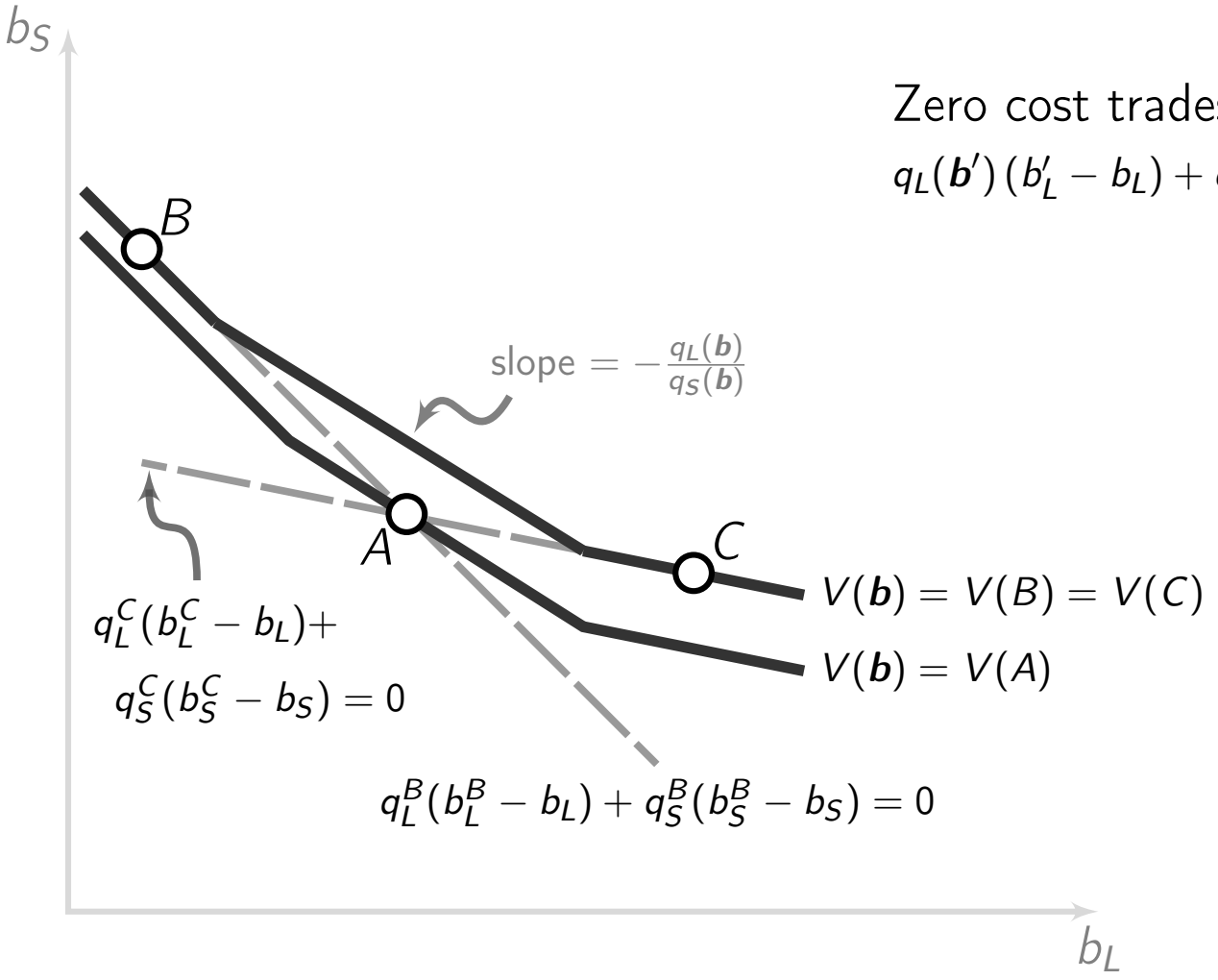

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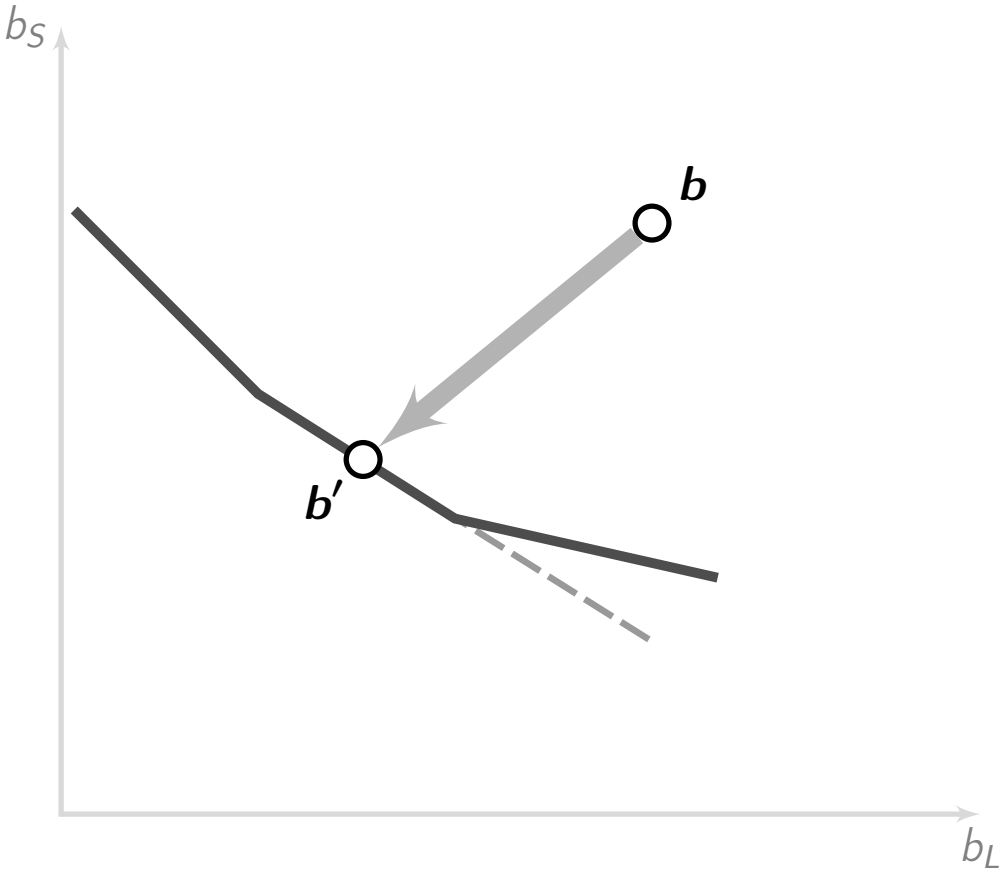
A Decomposition



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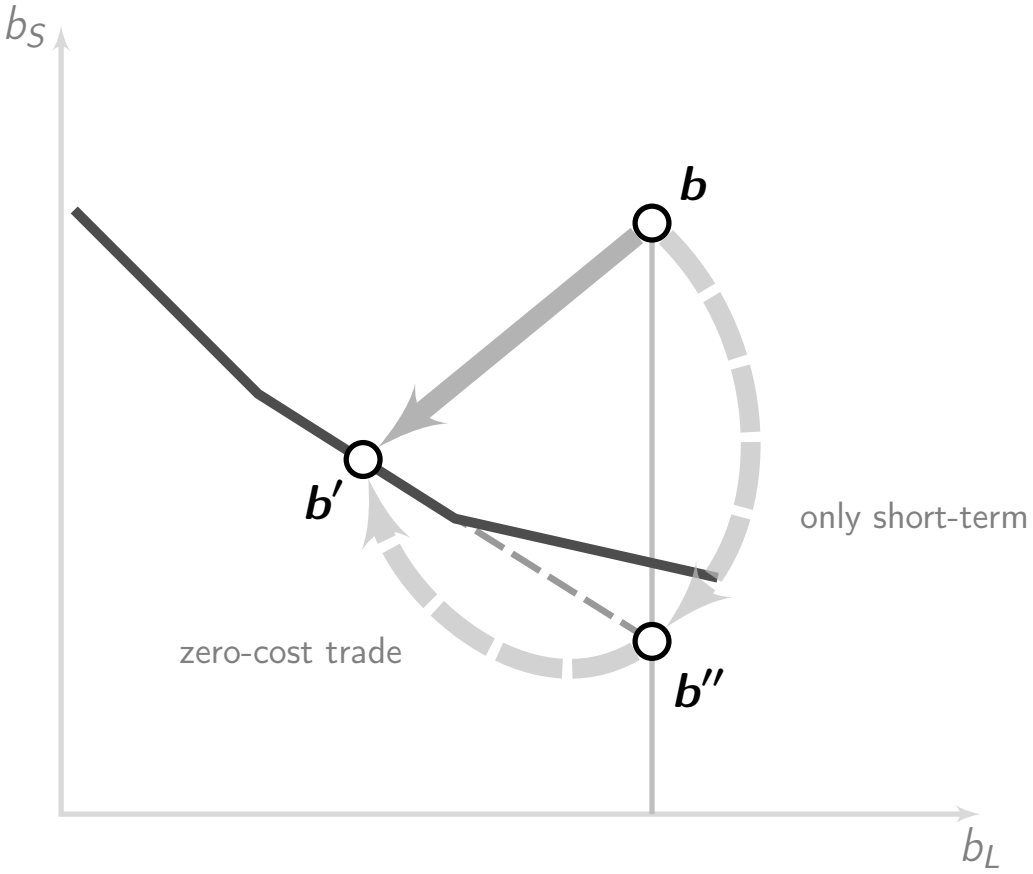


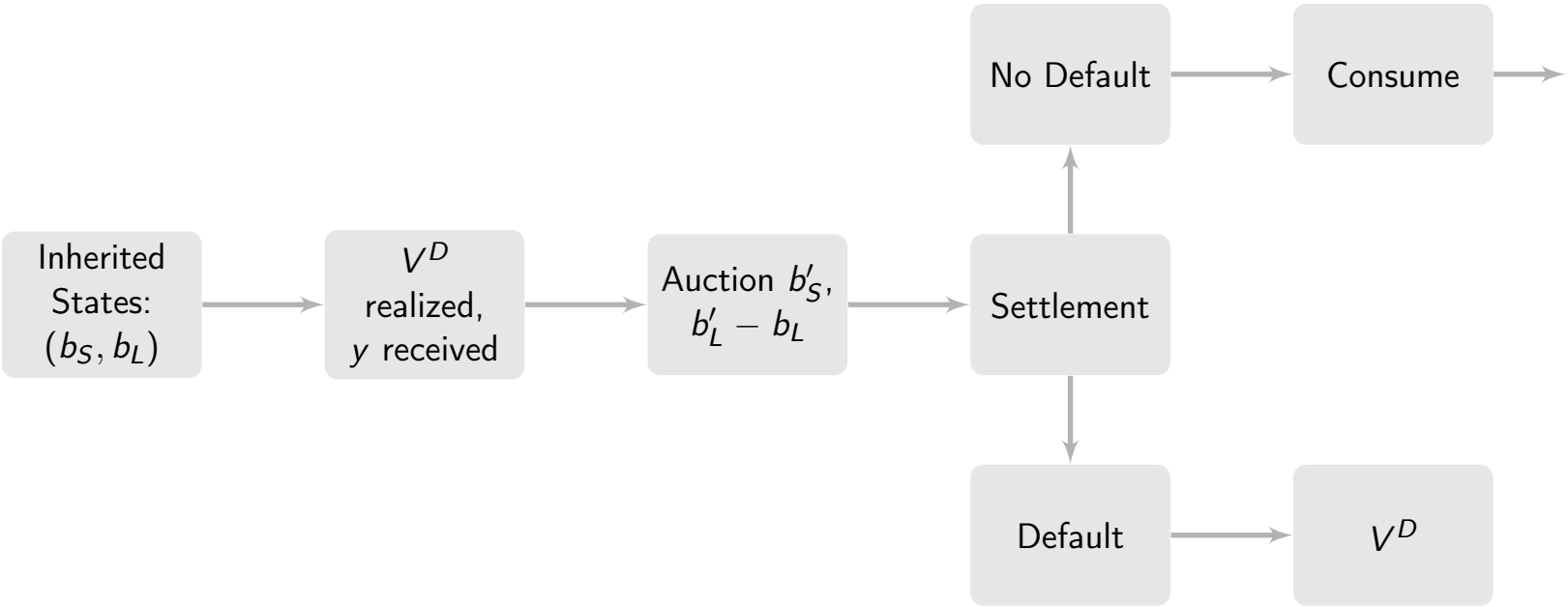
Buy Back Boondoggle

- Reminiscent of "Buy Back Boondoggle"
- Through incentives, rather than liquidation value
- Two-sided: Suboptimal to dilute long-term bondholders as well.

Extensions I: Rollover Risk and Coordination Failure

- Timing of benchmark $\Rightarrow$ Unique equilibrium
- No room for coordination failures or rollover risk
- Modify the timing

Cole-Kehoe Timing



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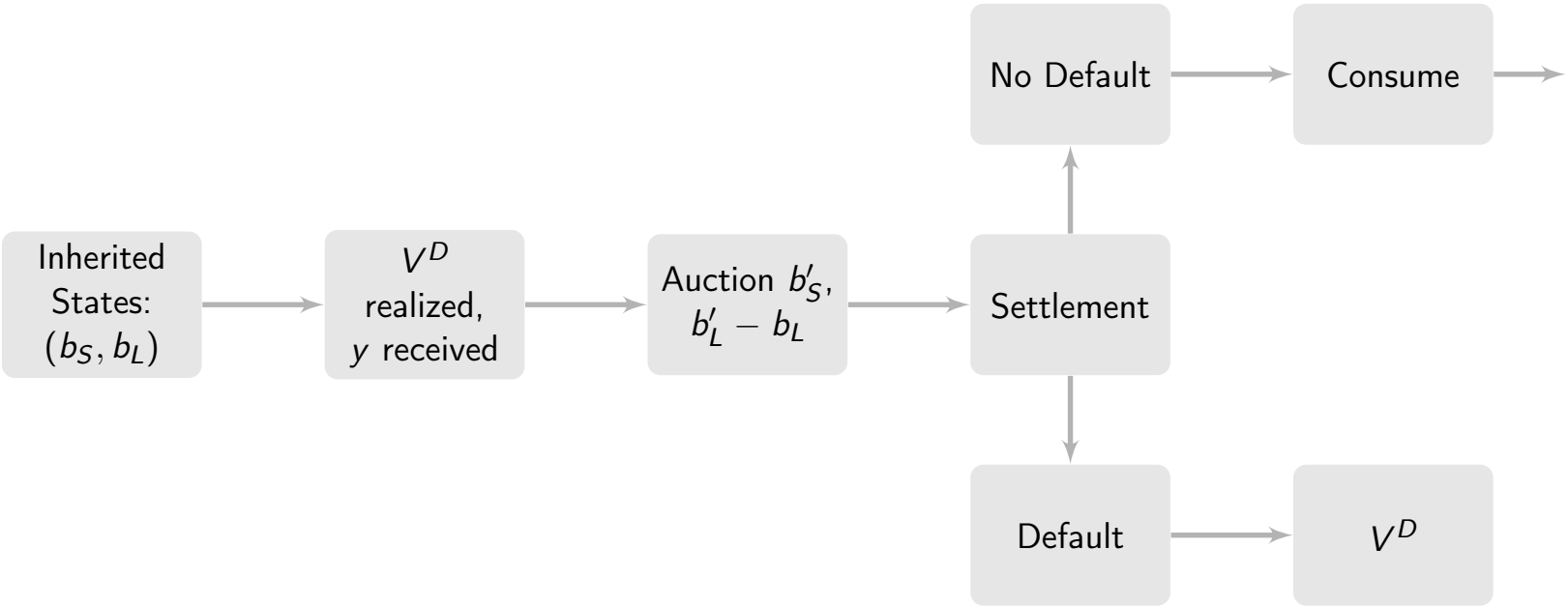

slight departure from CK: government doesn't get to keep the proceeds after default.

Markov Equilibrium in the new timing

New break even conditions

$$q_S(\boldsymbol{b}, V^D, \boldsymbol{b}') = (1 - D(\boldsymbol{b}, V^D, \boldsymbol{b'}))\mathbb{E}\left[1 - D' \middle| V^D\right]$$

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Markov Equilibrium

- Defined as before
- Multiplicity of equilibria: look for equilibria where the no-default area shrinks.

Coordination Failures

New No-Default Region

$$\tilde{\boldsymbol{ND}} = \left\{ \boldsymbol{b} \in \mathcal{B} \middle| u(y - (1+r)b_S - rb_L) + \beta \frac{u(y - rb_L)}{1-\beta} \geq \overline{V}^D \right\}.$$

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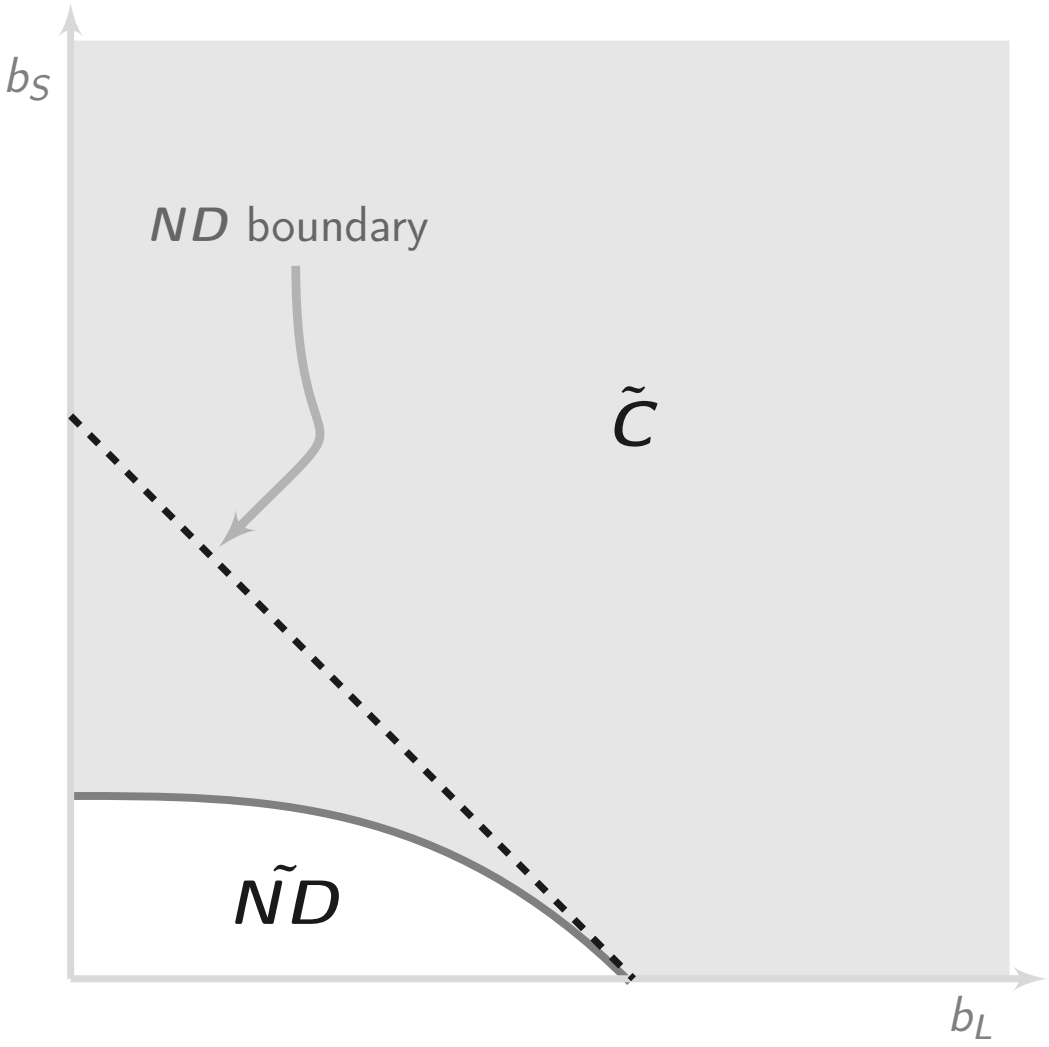
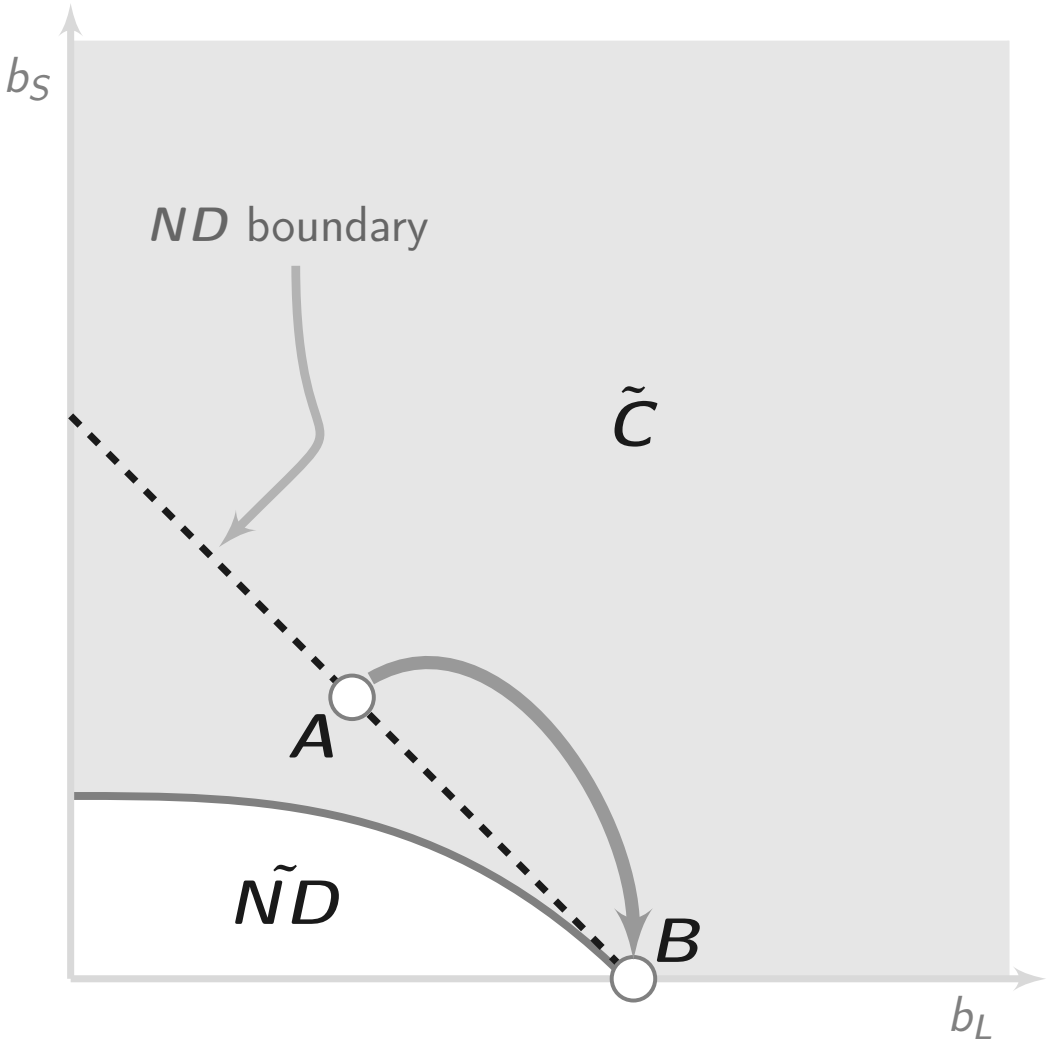


New No-Default Region



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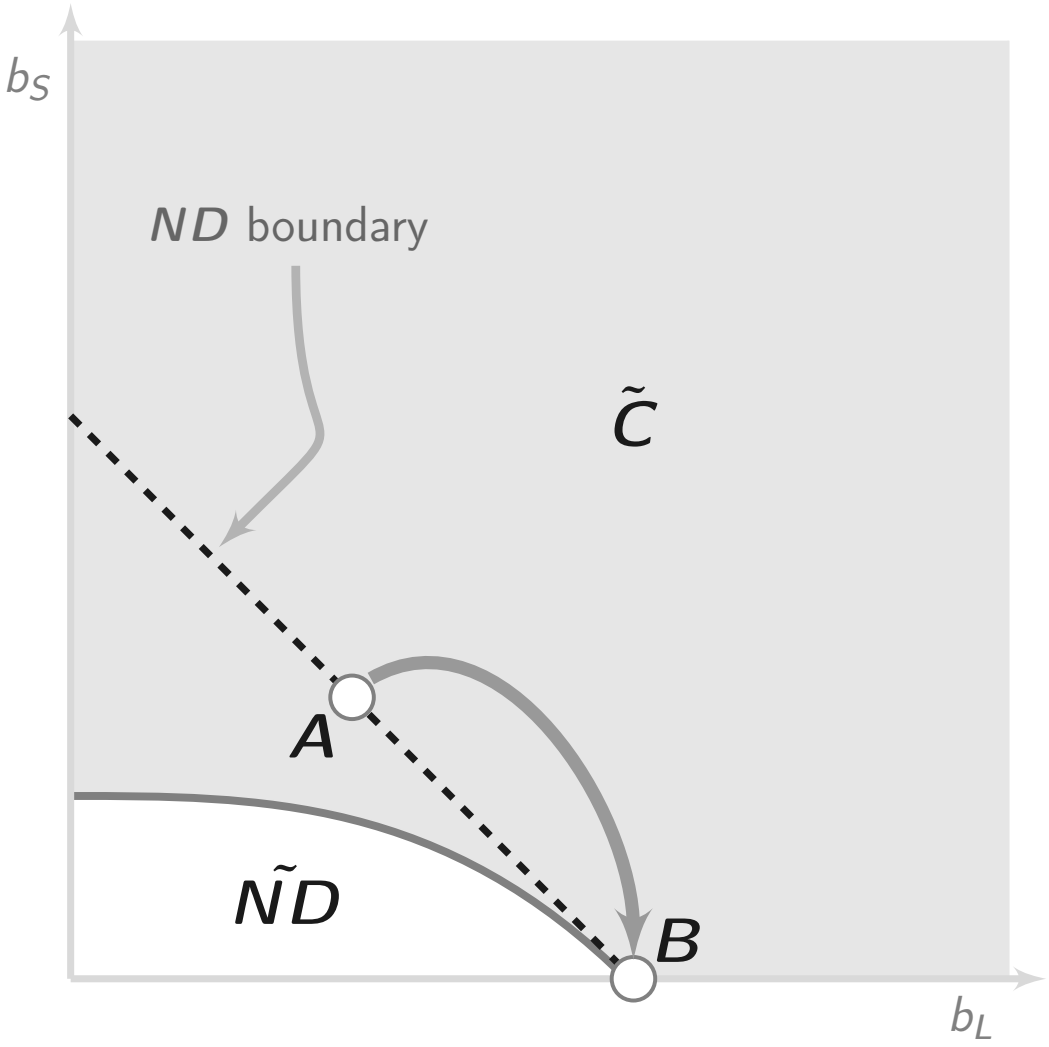
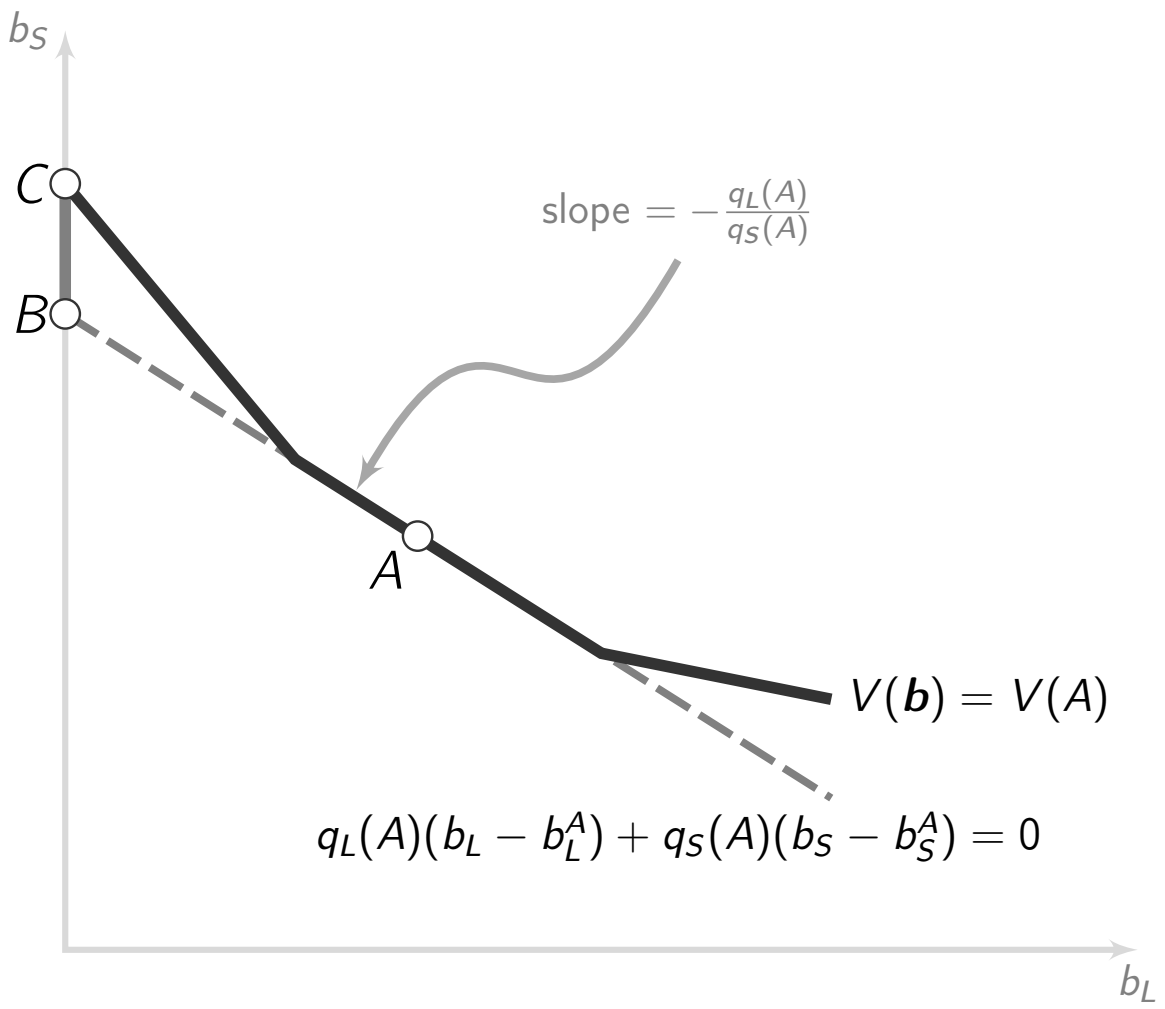


everything remains the same, including value functions

Coordination Failures

- Two sources of vulnerability
  - Fundamental (V^D)
  - Coordination Failure
- To eliminate both, need to pay down debt and extend maturity
  - If extend first, lose incentive to pay down debt
  - Prices make this sub-optimal in equilibrium
- Equilibrium sequencing is to deal with debt level first, and then adjust maturity

Restructuring



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- In our environment, maturity choice cannot be used to hedge risk
- Suppose need to raise g in an initial period but face a risk of regime change
- Ex ante environment is before realization of λ:

$$\tilde{\lambda} = \begin{cases} \lambda \text{ with probability } p \\ 0 \text{ with probability } 1 - p \end{cases}$$

- Trade-off incentives of short-term bonds with insurance of long-term bonds.
- Insurance favors long-term bond as lenders bear some of the risk of a high λ realization.
  - Costly due to the ex post incentives in bad regime

Conclusions

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- Strategies using long-term bonds during deleveraging are costly
  - even though bonds are price actuarially fair
  - no matter what you do, prices move against you