Take the Short Route
How to repay and restructure sovereign debt with multiple maturities

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Maturity Choice of Government Debt

Questions

▶ What is the equilibrium maturity management in a crisis?
Maturity Choice of Government Debt
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▶ Model focused on interplay of maturity, dynamics and risk of default
Maturity Choice of Government Debt

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▸ Why do sovereigns gravitate towards short-term debt when faced with the risk of default?

How

▸ Model focused on interplay of maturity, dynamics and risk of default

Sharp answer

▸ During deleveraging
  ▸ Do not issue or buy back long-term bonds – only use short term bonds

▸ Leave any active maturity management to the end

▸ Market outcome is Pareto inefficient
Context of Environment

Legacy Debt

- “Ex Ante” tranquil regime
- Debt raised when threat of default was low
- Maturity choice governed by traditional considerations (tax smoothing, spanning, providing benchmark assets)
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Crisis Regime

▶ Now face a significant risk of default – primary concern
▶ What to do about inherited legacy debt?
▶ Goal is to isolate and analyze the equilibrium response to this threat
Basic Environment

- Discrete time, infinite horizon, small open economy
- Government preferences: \( \sum_{t=0}^{\infty} \beta^t u(c_t) \)
- Risk-neutral lenders with opportunity cost \( R = 1 + r = \beta^{-1} \)
- Non-contingent bonds of different maturity
  - Benchmark: one-period \( (b_S) \) and perpetuity \( (b_L) \), with prices \( (q_S, q_L) \) and normalized coupons \( r \)
- Constant endowment \( y \)
Basic Environment

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  - Benchmark: one-period ($b_S$) and perpetuity ($b_L$), with prices ($q_S, q_L$) and normalized coupons $r$
- Constant endowment $y \leftrightarrow$ No Spanning Motive
Limited commitment

- Country can default. Outside option: $V^D$
- $V^D$ varies stochastically $\{\underline{V}^D, \overline{V}^D\}$. $\underline{V}^D < \overline{V}^D$.
- $\underline{V}^D$: strong enforcement regime – initial state
- Transition probability $\lambda$
- $\overline{V}^D$: weak enforcement regime – absorbing state (wlog)
Timing in the benchmark model

Inherited States: $(b_S, b_L)$

$V^D$ realized, $y$ received

No Default → Auction $b'_S$, $b'_L - b_L$, Payment $Rb_S + rb_L$ → Consume

Default

$V^D$
Towards a Markov Equilibrium

State

- \( b = (b_S, b_L), \quad \mathcal{B} = \mathbb{R}_+ \times \mathbb{R}_+ \)

Government’s problem if repay today:

\[
V(b, V^D) = \max_{\{c \geq 0, b \in \mathcal{B}\}} \left\{ u(c) + \beta \mathbb{E} \left[ \max \langle V(b', V^{D'}), V^{D'} \rangle | V^D \right] \right\}
\]

subject to:

\[
c + (1 + r)b_S + rb_L \leq y
\]

\[
+ q_S(V^D, b')b_S' + q_L(V^D, b')(b'_L - b_L).
\]

Default:

- \( D(b, V^D) = 1 \) if \( V^D > V(b, V^D) \), and zero otherwise
Investors’ break even conditions

\[ q_S(V^D, b') = \mathbb{E} \left[ 1 - D(b', V^{D'}) \middle| V^D \right] \]

\[ q_L(V^D, b') = \mathbb{E} \left[ \left( 1 - D(b', V^{D'}) \right) \left( \frac{r + q'_L}{1 + r} \right) \middle| V^D \right] \]
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Time Consistency

- Long-term bonds present a time consistency issue – price today depends on future debt policies

- Short-term bonds are not subject to this issue – at time of issuance, default risk over the maturity of the bond fully pinned down independent of future fiscal policies
Markov Equilibrium

Definition 1

A Markov Perfect Equilibrium consists of policy functions $C$, $B_S$, $B_L$, and $D$, and pricing schedules $q_S$ and $q_L$, such that for all debt positions $b \in B$ and $V^D \in \{ V^D, \overline{V}^D \}$, and absent a prior default:

(i) the policy functions $C$, $B_S$, and $B_L$, solve the government’s problem conditional on $q_S$ and $q_L$;

(ii) $D$ is an indicator function that takes one if $V(b, V^D) < V^D$ and zero otherwise; and

(iii) the creditors’ break-even conditions are satisfied with $q_i \in [0, 1]$, $i = S, L$, given the government’s policy functions.
Regions

- **No-Default**

\[
ND = \{ b \in B | V(b, V^D) \geq V^D, \forall V^D \}.
\]

- Absorbing state with risk-free prices: 

\[
V = \frac{u(y-r(b_S+b_L))}{1-\beta}
\]

- Boundary given by \( \bar{B} \):

\[
\frac{u(y-\bar{r}\bar{B})}{1-\beta} = \bar{V}^D
\]

- **ND** = \{\((b_S, b_L)|b_S + b_L \leq \bar{B}\}\).
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- \( ND = \{(b_S, b_L) \mid b_S + b_L \leq \bar{B} \} \).

- Crisis:

\[ C = \left\{ b \in B \mid V(b, \underline{V}^D) \geq \underline{V}^D \quad \& \quad V(b, \bar{V}^D) < \bar{V}^D \right\}. \]

- Default with probability \( \lambda \)
Regions

- **No-Default**

\[
ND = \{ b \in B \mid V(b, V^D) \geq V^D, \forall V^D \}.
\]

- Absorbing state with risk-free prices: 

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V = \frac{u(y-r(b_S+b_L))}{1-\beta}
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- Boundary given by \( \overline{B} \):

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\frac{u(y-r\overline{B})}{1-\beta} = \overline{V}^D
\]

- \( ND = \{(b_S, b_L)\mid b_S + b_L \leq \overline{B}\} \).

- **Crisis**:

\[
C = \left\{ b \in B \mid V(b, \overline{V}^D) \geq \overline{V}^D \; \& \; V(b, \overline{V}^D) < \overline{V}^D \right\}.
\]

- Default with probability \( \lambda \)

- **Default**: 

\[
D = \{ b \mid V(b, \overline{V}^D) < \overline{V}^D \}.
\]
$B = b_S + b_L$

$V(b_S, b_L) < V^D$

$V^D \leq V(b_S, b_L) < \overline{V}^D$

$ND$

$C$

$\overline{V}^D = \frac{u(y - r(b_S + b_L))}{1 - \beta}$

$V(b_S, b_L) \geq \overline{V}^D$
Markov Equilibrium Prices

Short-term Bonds

\[ q_S(b) = \mathbb{E} \left[ 1 - D(b', V^D) \middle| V^D \right] = \begin{cases} 1 & \text{if } b \in ND \\ 1 - \lambda & \text{if } b \in C \\ 0 & \text{if } b \in D \end{cases} \]

Long-term Bonds

\[ q_L(V^D, b') = \mathbb{E} \left[ \left( 1 - D(b', V^{D'}) \right) \left( \frac{r + q'_L}{1 + r} \right) \middle| V^D \right] = r \sum_{t=1}^{T(b)} \left( \frac{1 - \lambda}{1 + r} \right)^t + \left( \frac{1 - \lambda}{1 + r} \right)^{T(b)} \]

- Where \( T(b) \in \{0, 1, ..., \infty\} \) : equilibrium exit time from \( C \)
Main Results

Result 1

- A weakly optimal equilibrium policy is to never issue or repurchase long-term bonds ($b'_{L} = b_{L}$)
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- A weakly optimal equilibrium policy is to never issue or repurchase long-term bonds ($b'_L = b_L$)

Result 2
- If $T(b) < \infty$, this policy is strictly optimal
Outline of Proof for Result 1

- Pick an arbitrary (feasible) exit time $T$ and substitute associated prices into budget set

- Any consumption stream that satisfies this budget set can be implemented using only short-term bonds (paying only the coupon on perpetuities)
  
  - As this is true for any feasible $T$, it holds for the equilibrium $T(b)$ as well, and thus the equilibrium consumption path can always be implemented trading only short-term bonds

- Equilibrium $T(b)$ is simply the max utility over all possible exit times implemented by trading short-term bonds
Intuition for Result 1

- Prices of short-term bonds are determined by outstanding debt at time of auction.

- No time-consistency issue – government can vary $T$ without changing short-term bond prices conditional on debt.

- Short-term bond prices are actuarially fair for any $T$ and thus the exit time that maximizes welfare is an (the) equilibrium.
The dummy maximization problem

- Suppose the government could commit to an exit-time $T$
- but restricted to use only short-term bonds
The dummy maximization problem

- Suppose the government could commit to an exit-time $T$
- but restricted to use only short-term bonds

$$W(b, T) = \max_{\{b_S, T, \{c_t\}_{t=0}^{T-1}\}} \left\{ \sum_{t=0}^{T-1} \beta^t (1 - \lambda)^t u(c_t) + \beta^T (1 - \lambda)^{T-1} \times \left( \frac{u(y - r(b_S, T + b_L))}{1 - \beta} \right) + \sum_{t=1}^{T-1} \beta^t (1 - \lambda)^{t-1} \lambda \overline{V}^D \right\}$$

subject to:

$$b_S \leq (1 + r)^{-1} \left( \sum_{t=0}^{T-1} \left( \frac{1 - \lambda}{1 + r} \right)^t (y - c_t - rb_L) + \left( \frac{1 - \lambda}{1 + r} \right)^{T-1} b_S, T \right)$$

$$b_{S, T} \leq \overline{B} - b_L.$$
Main result #1: Two Lemmas

- Lemma 1: $V(b, V^D) \leq W(b, T(b))$
- Lemma 2: $V(b, V^D) \geq W(b, T)$ for any $T$
- $\Rightarrow V(b, V^D) = \sup_T W(b, T)$
Alternative Maximization Problem

- Can solve maximization problem of paying down short-term bonds to exit in arbitrary period $T$: $W(b, T)$

- $V(b) = \sup_T W(b, T)$

- Can solve government’s problem without knowing long-term prices, and then with $T(b)$ we have $q_L$

- Avoids complicated fixed point problem on prices and policies

- Equilibrium is unique up to integer constraint on $T(b)$
At point A: $V(b, V^D) < \overline{V}^D$
Intermission – Why save?

Stay put at A: \( T = \infty \)

\[ c^\infty = y - r(b_S + b_L) - \lambda b_S \]
Intermission – Why save?

Stay put at A: $T = \infty$

$c^\infty = y - r(b_S + b_L) - \lambda b_S$

Go to ND in one step: $T = 1$

$c^1 = y - r(b_S + b_L) - (b_S + b_L - \bar{B})$
Intermission – Why save?

Stay put at A: \( T = \infty \)
\[
c^\infty = y - r(b_S + b_L) - \lambda b_S
\]

Go to ND in one step: \( T = 1 \)
\[
c^1 = y - r(b_S + b_L) - (b_S + b_L - \bar{B})
\]
\[
\lambda b_S \geq b_S + b_L - \bar{B}
\]

- As \( b_S + b_L \to \bar{B} \), strictly prefer to exit if \( b_S \gg 0 \)
Intermission – Why save?

Stay put at A: \( T = \infty \)
\[ c^\infty = y - r(b_S + b_L) - \lambda b_S \]

Go to ND in one step: \( T = 1 \)
\[ c^1 = y - r(b_S + b_L) - (b_S + b_L - \bar{B}) \]
\[ \lambda b_S \geq b_S + b_L - \bar{B} \]

- As \( b_S + b_L \to \bar{B} \), strictly prefer to exit if \( b_S >> 0 \)
- Default is not a zero-sum game
- Short-term debt puts the full cost of potential default on the sovereign
Equilibrium Iso-T regions

Equilibrium Iso-T regions

$T(b) = 1$

$T(b) = 2$

$T(b) = 3$

$T(b) = \infty$

$\frac{ql(b)}{qs(b)}$

$ND$

Speed of exit affected by maturity composition
More long term bonds: slower exit
Equilibrium Iso-T regions

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Speed of exit affected by maturity composition
More long term bonds: slower exit
Hopenhayn and Werning consider an optimal contract between a principal and agent in this framework (with unobservable outside option shocks)

Show that it can be implemented using short-term bonds

Did not consider legacy debt or multiple principals, but extension implies our result is more general than our incomplete contracting space –> the four of us are working on this now so stay tuned

Raises the question about why not dilute (or buy back) previous bond holders?
Equilibrium Iso-V curves

\( b_{S} \)

\( b_{L} \)
Equilibrium Iso-V curves

slope = $-\frac{q_L(b)}{q_s(b)}$
Equilibrium Iso-V curves

\[ \text{slope} = -\frac{q_L(b)}{q_S(b)} \]
Equilibrium Iso-V curves

Zero cost trades
\[ q_L(b')(b'_L - b_L) + q_S(b')(b'_S - b_S) = 0 \]
Equilibrium Iso-V curves

Zero cost trades

\[ q_L(b')(b'_L - b_L) + q_S(b')(b'_S - b_S) = 0 \]

\[ V(b) = V(B) \]

\[ V(b) = V(A) \]

\[ q_L^B(b_L^B - b_L) + q_S^B(b_S^B - b_S) = 0 \]
Equilibrium Iso-V curves

Zero cost trades

\[ q_L(b')(b'_L - b_L) + q_S(b')(b'_S - b_S) = 0 \]
A Decomposition

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zero-cost trade

only short-term
Buy Back Boondoggle

- Reminiscent of “Buy Back Boondoggle”
- Through incentives, rather than liquidation value
- Two-sided: Suboptimal to dilute long-term bondholders as well.
Extensions I: Rollover Risk and Coordination Failure

- Timing of benchmark ⇒ Unique equilibrium
- No room for coordination failures or rollover risk
- Modify the timing
Cole-Kehoe Timing

Inherited States: $(b_S, b_L)$

$V^D$ realized, $y$ received

Auction $b'_S$, $b'_L - b_L$

Settlement

No Default

Default

Consume

Slight departure from CK: government doesn't get to keep the proceeds after default.
Cole-Kehoe Timing

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Markov Equilibrium in the new timing

New break even conditions

\[ q_S(b, V^D, b') = (1 - D(b, V^D, b')) \mathbb{E} \left[ 1 - D' \left| V^D \right. \right] \]

\[ q_L(b, V^D, b') = (1 - D(b, V^D, b')) \mathbb{E} \left[ (1 - D') \left( \frac{r + q'_L}{1 + r} \right) \left| V^D \right. \right] \]
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\]

Markov Equilibrium

- Defined as before

- Multiplicity of equilibria: look for equilibria where the no-default area shrinks.
Coordination Failures

New No-Default Region

\[ \tilde{ND} = \left\{ b \in B \mid u(y - (1 + r)b_s - rb_L) + \beta \frac{u(y - rb_L)}{1 - \beta} \geq V^D \right\}. \]
New No-Default Region

$ND$ boundary

$\tilde{C}$

$\tilde{ND}$
New No-Default Region

\[ \tilde{C} \]

ND boundary

\[ \tilde{ND} \]

Everything remains the same, including value functions.
New No-Default Region

everything remains the same, including value functions
Coordination Failures

- Two sources of vulnerability
  - Fundamental ($V^D$)
  - Coordination Failure

- To eliminate both, need to pay down debt and extend maturity
  - If extend first, lose incentive to pay down debt
  - Prices make this sub-optimal in equilibrium

- Equilibrium sequencing is to deal with debt level first, and then adjust maturity
Restructuring

\[ V(b) = V(A) \]

\[ q_L(A)(b_L - b_L^A) + q_S(A)(b_S - b_S^A) = 0 \]

slope \( = -\frac{q_L(A)}{q_S(A)} \)
More general portfolio

- Perpetuities and one-period bonds
- More general maturities?
More general portfolio

- Perpetuities and one-period bonds
- More general maturities?

- Main result can be generalized as long as there is a one-period bond.
  - Use only short-term assets while deleveraging
  - Pay coupons and retire long-term bonds as they mature

Important: Result is about issuances not stocks

While deleveraging, issuances should be short term

Whether the outstanding stock is shifting its maturity composition depends on maturity length and speed of deleveraging

Motivating facts were about issuances.
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What about hedging?

- In our environment, maturity choice cannot be used to hedge risk.
What about hedging?

- In our environment, maturity choice cannot be used to hedge risk.
- Suppose need to raise $g$ in an initial period but face a risk of regime change.
- Ex ante environment is before realization of $\lambda$:
  \[ \tilde{\lambda} = \begin{cases} 
  \lambda \text{ with probability } p \\
  0 \text{ with probability } 1 - p
  \end{cases} \]
- Trade-off incentives of short-term bonds with insurance of long-term bonds.
- Insurance favors long-term bond as lenders bear some of the risk of a high $\lambda$ realization.
  - Costly due to the ex post incentives in bad regime.
Conclusions

- Sharp results about maturity choice
Conclusions

- Sharp results about maturity choice

- Strategies using long-term bonds during deleveraging are costly
  - even though bonds are price actuarially fair
  - no matter what you do, prices move against you