Macroprudential Regulation Versus Mopping Up After the Crash*

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Abstract

A growing literature has investigated optimal policy responses to the externalities that arise from financial crises. Some have argued in favor of macro-prudential regulation to mitigate crisis risk ex-ante, whereas others propose that ex-post stimulus measures are more desirable. We show that the optimal policy mix in the face of financial instability consists of a combination of ex-ante macro-prudential and ex-post stimulus measures, as both forms of intervention generally impose costs that are second-order (i.e. negligible for small amounts but increasing in a convex fashion).

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1 Introduction

A growing literature has argued in favor of macroprudential regulation based on models that interpret financial crises as episodes of financial amplification, i.e. in which the economy experiences a feedback loop of adverse price movements (in asset prices or exchange rates) and tightening financial constraints. As pointed out in Gromb and Vayanos (2002) and Jeanne and Korinek (2011), financial amplification

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effects involve pecuniary externalities because atomistic agents do not internalize that their individual actions lead to relative price movements that reinforce shocks in the aggregate.

However, there has been an intense debate about the relative desirability of prudential measures that attempt to curb indebtedness ex-ante, before crises materialize, and policy measures that are taken ex-post, once a crisis has hit. This is probably best exemplified by the so-called “Greenspan doctrine” (see Greenspan, 2002, 2011; Blinder and Reis, 2005), according to which ex-ante intervention to prevent booms are too costly compared to “mopping up” measures after a financial crisis has materialized.

This paper studies the desirability of different forms of policy interventions in a stylized but general model of financial crises, which we describe as episodes of financial amplification in the spirit of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). We first describe four settings in which a planner can restore the first-best equilibrium: (i) if a planner can disregard financial constraints, (ii) if a planner can engage in lump-sum transfers between borrowers and lenders, (iii) if a planner has superior commitment and enforcement powers that allow her to circumvent the constraint and (iv) if a planner can subsidize asset prices using revenue derived from a lump-sum tax. In practice none of these conditions is likely to be met.

Next we turn our focus to second-best policy measures. We first solve the problem of a constrained social planner who has to obey the same financial constraint as decentralized agents, but who – unlike competitive agents – internalizes the effects of her actions on aggregate prices. Such a planner reduces borrowing in unconstrained times so as to mitigate the amplification effects in constrained times. This could be implemented for example via a “macroprudential” tax on borrowing. We observe that this action can be interpreted as a policy according to the theory of the second-best: the planner’s intervention in unconstrained times introduces a second-order cost, but the relaxation of binding constraints results in a first-order benefit.

According to the theory of the second best (Lipsey and Lancaster, 1956), it is generally desirable to intervene along all available dimensions when engaging in second-best policies. We therefore extend our analysis to include several policy measures that are complementary to macroprudential taxation, including asset price support and labor stimulus policies. We show that it is generally desirable to engage in a mix of all these policies. The optimal policy mix consists of a combination of all measures available such that the marginal cost of each intervention equals its expected marginal benefit.

When we consider policy measures that involve intervention in the period that binding constraints arise rather than ex-ante intervention in the period before, a time consistency problem arises: a planner would like to commit to be “tough” and not mitigate binding constraints so as to induce private agents to engage in the effi-
cient level of precautionary savings given the pecuniary externality. However, once a crisis materializes, the planner finds it optimal to renege on her commitment and provide relief to constrained borrowers. We show that imposing macroprudential regulation gives the policymaker a tool to correct any distortions to ex-ante behavior and implement the optimal level of precautionary savings. Therefore macroprudential regulation alleviates the time consistency problems associated with ex-post intervention.

All the second-best policies that we study work by triggering pecuniary externalities, i.e. by affecting aggregate prices. Whereas competitive agents take prices as given, a planner internalizes that she can affect prices in the economy, for example by steering individual agents towards borrowing less, increasing their asset demand, or supplying more labor. Pecuniary externalities are sometimes viewed as esoteric, but they are the driving force of financial amplification effects, which have been perceived to be at the center stage of many recent financial crises: falling asset prices and the resulting balance sheet effects have played a crucial role. See e.g. Brunnermeier (2009) or Adrian and Shin (2010) for a discussion of the role of financial amplification effects in the Global Financial Crisis of 2008/09, or Krugman (1999) and Mendoza (2002) for their role in the emerging market crises of the past two decades.

Under complete markets, the welfare theorems imply that pecuniary externalities do not matter. Under complete markets, the marginal rates of substitution of all agents in the economy are equal to the relative market prices for all goods. In such a setting the welfare theorems imply that policy intervention to manipulate relative prices introduces distortions that reduce welfare.

By contrast, during episodes of financial amplification, markets are incomplete and some agents face binding financial constraints and therefore value resources relatively more than unconstrained agents. A relative price movement that redistributes resources can therefore achieve a Pareto improvement, as shown e.g. in Stiglitz (1983) or Geanakoplos and Polemarchakis (1986). Gromb and Vayanos (2002), Lorenzoni (2008) and Farhi et al. (2009) have applied this to models of financial constraints. In the context of models of financial amplification such as Korinek (2007, 2010), the observation that there are “adverse balance sheet effects” during financial crises precisely captures that the redistributions that result from relative price movements matter greatly.

This implies that policymakers can improve welfare by instructing private agents to reduce the probability and severity of experiencing binding financial constraints. They can achieve this by manipulating the privately optimal decisions of agents in a way that affects relative prices so as to relax such constraints. Lorenzoni (2008) shows that there is generally excessive borrowing and investment in such a setting, and Korinek (2010) finds that agents will not engage in sufficient insurance against adverse shocks that trigger financial amplification, even if state-contingent financial
instruments are available. Jeanne and Korinek (2010ab, 2011) and Bianchi and Mendoza (2011) argue that total borrowing should be reduced if uncontingent debt is the only financial instrument. All these papers have in common that they focus on ex-ante or “macro-prudential” measures to reduce the risk of experiencing financial amplification effects.

Benigno, Chen, Otrok, Rebucci and Young (2009, 2010a, 2010b) study the desirability of ex-post intervention in emerging economies that experience financial amplification effects. Benigno et al. (2009) study how a planner can use direct exchange rate intervention to mitigate adverse balance sheet effects. Benigno et al. (2010ab) argue that a planner can appreciate the real exchange rate by shifting labor out of the non-tradable sector and into the tradable sector to mitigate adverse balance sheet effects. Since these interventions relax financial constraints, the authors find that the economy can in equilibrium sustain a higher quantity of borrowing. We agree with their positive finding on the quantity of borrowing, but not with the interpretation that it makes macroprudential taxation undesirable or unrobust. As was pointed out e.g. by Atkinson and Stiglitz (1980), when a planner has multiple policy instruments, it is essential to derive optimal Ramsey taxes in order to provide policy advice – a simple comparison of equilibrium quantities can be highly misleading.\footnote{A simple comparison captures the basic intuition of our result on prudential measures versus ex post intervention: If the introduction of airbags has reduced the expected death toll of car accidents (which is an ex-post device to reduce the cost of crashes), it may—at the margin—be optimal for drivers to be less careful ex-ante and drive at higher speeds. However, this comparison across two different regimes does not imply that it is optimal to abolish speeding regulations and instead subsidize reckless driving.}

None of the papers in the described literature explicitly derives the optimal mix of Ramsey taxes/subsidies if a planner has access to both ex-ante and ex-post policy measures. Our paper fills the gap. We find that it is optimal to use both ex-ante prudential measures and ex-post interventions, and the optimal policy mix is such that the marginal cost/benefit ratio of ex-ante measures equals the marginal cost/benefit ratio of ex-post measures. Even though we show that it is always optimal to impose a tax on borrowing, we also replicate the finding of Benigno et al. that the equilibrium quantity of debt may be either higher or lower in an economy where policymakers have access to ex-post interventions.

2 Benchmark Model of Financial Amplification

2.1 Setup

Our benchmark is a small open economy in a one-good world with three time periods $t = 0, 1, 2$. The economy is populated by a continuum of atomistic identical
consumers who consume \( c_t \) every period. We denote their utility as

\[
U = u(c_0) + u(c_1) + c_2.
\]  

(1)

where \( u(c_t) = c_t^{1+\gamma} / (1 + \gamma) \) is a CRRA utility function. Consumers derive income from two sources: in period 1 they obtain a stochastic endowment \( \tilde{e} \) which is not pledgeable to creditors. The endowment depends on the state of nature \( \omega \in \Omega \). (In an extension, we will replace the endowment by labor income.) Furthermore, consumers are born with \( \theta_1 = 1 \) units of an asset that yields a payoff \( y \geq 2 \) in period 2, which we assume deterministic for simplicity. The asset can be pledged as collateral, but at the end of each period it has to be held by domestic consumers, otherwise it looses its value. This captures in a simplified way the notion of Kiyotaki and Moore (1997) that domestic consumers have a higher use for the domestic asset than other agents. Domestic consumers can buy or sell the asset in a perfectly competitive domestic market at price \( p_1 \) at the end of period 1.

The domestic consumer issues debt \( d_0 \) and \( d_1 \) in periods 0 and 1 and repays in period 2. Debt is bought by international investors who have a zero discount rate. We assume that domestic consumers and international investors cannot enter insurance contracts. A potential motivation for this is that the risky consumer endowment \( \tilde{e} \) may not be verifiable. In equilibrium there is no default and consumers can borrow at investors’ discount rate of zero. The resulting budget constraints are

\[
\begin{align*}
c_0 &= d_0 \\
c_1 &= d_1 + \tilde{e} + (\theta_1 - \theta_2) p_1 \\
c_2 &= \theta_2 y - d_0 - d_1
\end{align*}
\]

(2)

We assume that consumers may threaten to default and renegotiate their debts both in period 1 and in period 2. If a consumer defaults, lenders can seize up to a fraction \( \phi \) of his asset holdings, which they have to re-sell to other consumers at the prevailing market price – otherwise the assets would become worthless. For simplicity assume that the insider has all the bargaining power. To be renegotiation-proof in period 1, the debt level of insiders then has to satisfy

\[
d_0 + d_1 \leq \phi \theta_1 p_1
\]

(3)

To be renegotiation-proof in period 2, an analogous equation has to hold for the period 2 asset price. In equilibrium, we can show that \( p_1 \leq p_2 \) and this second constraint can be omitted. In other words, the incentive to renegotiate is highest at the end of period 1 when asset prices may be depressed because of binding constraints. The possibility to renegotiate in period 1 is essential to introduce amplification effects in our model. This captures the notion that a fall in asset prices, even if temporary, leads to “illiquidity.”

\[2\]If we drop constraint (3), the economy no longer exhibits the feedback loop from falling asset prices to tightening financial constraints.
The optimization problem of consumers is to choose \((c_0, c_1, c_2, d_0, d_1, \theta_2)\) to maximize (1) subject to the budget constraints (2) and the collateral constraint (3) to which we assign a shadow price \(\lambda\). The consumer’s relevant first-order conditions are

\[
\begin{align*}
u'(c_0) &= E[u'(c_1)] \\
u'(c_1) &= 1 + \lambda \\
p_1 &= \frac{y}{u'(c_1)} = yc_1^\gamma =: p(c_1)
\end{align*}
\]

The first two are the consumer’s Euler equations, and the last one is the asset pricing condition, which defines an asset price function \(p(c_1)\) that satisfies \(p'(c_1) > 0\), i.e. the asset price is an increasing function of period 1 consumption.

### 2.2 Decentralized Equilibrium

In a symmetric decentralized equilibrium \(\theta_2 = 1\). We solve the problem through backward induction, focusing first at the equilibrium in periods 1 and 2. We denote the financial net worth of a consumer at the beginning of period 1 by \(m = \bar{e} - d_0\). If the collateral constraint in period 1 is loose so \(\lambda = 0\), the first-best level of consumption \(c^* = 1\) satisfying \(u'(c^*) = 1\) can be implemented, and the asset price satisfies \(p_1 = y\). The consumer’s period 1 financial net worth is sufficient to implement this equilibrium if and only if

\[
m \geq \hat{m} := c^* - \phi y \tag{4}
\]

If \(m\) is below this threshold, then consumers borrow the maximum amount possible \(\phi p(c_1)\) in period 1. Equilibrium is therefore determined as the solution to the implicit equation

\[
c_1 = \min \{c^*, m + \phi p(c_1)\} \tag{5}
\]

The two sides of this equation are illustrated in figure. In the constrained region, changes in net worth lead to an amplified response of consumption: Assume a marginal increase in net worth \(dm\). The direct effect of this is to allow consumers to raise consumption by \(dm\). However, in general equilibrium this pushes up the asset price by \(dm \cdot p'\), which relaxes the borrowing constraint by \(dm \cdot \phi p'\) and leads to further increases in consumption, asset prices and borrowing. The total effect of a marginal change in wealth is

\[
\frac{dc_1}{dm} = 1 + \phi p' + (\phi p')^2 + \ldots = \frac{1}{1 - \phi p'(c_1)} > 1
\]

Figure 1 illustrates these amplification effects by the dashed lines that represent a perturbation \(\pm \Delta m\) around the initial level of \(m\).
If \( m > 0 \) and \( \phi p' (c_1) < 1 \forall c_1 \), then equation (5) has a unique solution and defines a function \( c_1 (m) \) that is strictly increasing in \( (0, \hat{m}) \) and that takes the value \( c_1 (m) = c^* = 1 \) for \( m \geq \hat{m} \). Period 2 consumption is then given by \( c_2 = m + y - c_1 (m) \). We impose the following assumption to ensure the uniqueness of equilibrium:

**Assumption 1 (Uniqueness)** *The parameters satisfy \( \gamma \geq 1 \) and \( \phi < \frac{1}{\gamma y} \).*

Under these assumptions, the slope of the price function \( p' (c_1) = \gamma y c_1^{\gamma - 1} \) is maximized at the highest possible consumption level \( c^* = 1 \). As long as \( \phi < \frac{1}{\gamma y} \), the condition \( \phi p' (c_1) < 1 \) holds for any level of financial net worth \( m \).

Having solved for the period 1 and 2 equilibrium, the consumer determines his period 0 level of consumption \( c_0 = d_0 \) as the solution to the Euler equation

\[
u' (d_0) = E [u' (c_1 (\hat{e} - d_0))]\]

### 3 First-Best Equilibrium

To describe the benchmark for social efficiency, we solve for the first-best allocation of the economy, which maximizes the welfare of consumers (1) subject to the budget constraints (2), but, in the absence of the collateral constraint.

**Proposition 1 (First-Best Allocation)** *The first-best allocation of the economy satisfies \( c_{0b}^{fb} = c_1^{fb} = d_{0b}^{fb} = c^* \), \( d_1^{fb} = c^* - \hat{e} \) and \( c_2^{fb} = y - d_{0b}^{fb} - d_{1b}^{fb} \forall \omega \in \Omega \). It can be implemented if any of the following conditions is met:*
1. if a planner is not bound by the collateral constraint (first-best planning problem), or

2. if a planner can engage in lump-sum transfers that make the collateral constraint irrelevant (transfer policy), or

3. if a planner has superior powers of commitment and enforcement (crisis lending), or

4. if a planner can subsidize the asset price using revenue that has been raised in a non-distortionary manner to make the collateral constraint loose (asset price support).

Proof. We obtain the first-best allocations in the economy by following along the steps of the decentralized equilibrium under the assumption that the collateral constraint is loose. The different ways of implementing the first-best equilibrium work as follows:

1. The definition of a first-best planner is that she is bound only by the resource constraints and the technology of the economy and can ignore other constraints, such as the collateral constraint.

2. If the planner can engage in lump-sum transfers, then financial constraints are irrelevant and she can replicated the first-best equilibrium. The planner would transfer $d_{0}^{lb}$ and $d_{1}^{lb}$ from lenders to domestic borrowers in periods 0 and 1 and would transfer the amount $d_{0}^{lb} + d_{1}^{lb}$ back to lenders in period 2.

3. The planner can replicate this policy via “crisis lending” if she has superior powers of commitment and enforcement. Specifically, this requires that the planner can borrow $d_{0}^{lb}$ and $d_{1}^{lb}$ from lenders and commit to repay without offering collateral, lend on these resources to constrained consumers who do not have sufficient collateral, and enforce repayment from them.

4. Alternatively, assume the planner can subsidize asset holdings, say at rate $s_{\theta}$, and raise the required revenue through lump-sum taxation. This modifies the asset-pricing equation to

$$p(c_{1}, s_{\theta}) = yc_{1}^{r} + s_{\theta}$$

The planner ensures that the collateral constraint is loose and the first-best level of consumption is feasible by imposing a subsidy rate that satisfies

$$\phi s_{\theta} \geq c^{*} - m - \phi y$$
Discussion  Any of the four conditions described are sufficient to return the economy to its first-best equilibrium. If the first-best equilibrium is achieved, no financial amplification effects occur and no further policy intervention is desirable. All price effects – including pecuniary externalities – are irrelevant.

This result continues to hold if we extend our benchmark model to allow for endogenous investment but give the planner an instrument to affect the level of investment. In such a setting, a subsidy to asset holdings would create incentives for over-investment, but a planner with a full set of policy instruments could simultaneously subsidize asset holdings and tax asset creation so as to restore the first-best equilibrium.

4 Second-Best Planning Problems

In this section, we assume that the first-best policies described in the previous section are not available. This captures the realistic notion that policy intervention generally involves trade-offs. We first describe problem of a constrained social planner in the described economy, who has access to the same instruments and is subject to the same constraints as private agents. We show that such a planner can improve on the decentralized equilibrium when borrowing constraints are binding because she internalizes pecuniary externalities, whereas decentralized agents take prices as given. Then we generalize our analysis to introduce additional second-best instruments. This allows us to analyze the trade-off between ex-ante prudential regulation and ex-post intervention.

4.1 Constrained Planning Problem

We define the constrained optimum as the equilibrium chosen by a planner who cannot directly set prices, but who picks real allocations \((c_0, c_1, d_0, d_1, \theta_2)\) to maximize (1) subject to (2) and (3), while respecting that the asset price is determined by the intertemporal marginal rate of substitution of private agents, as expressed in equation (??). The planner’s resulting first-order conditions are

\[
\begin{align*}
\text{FOC} (d_0) : \quad & u'(c_0) = E [1 + \lambda] \\
\text{FOC} (d_1) : \quad & u'(c_1) = 1 + \lambda (1 - \phi p'(c_1))
\end{align*}
\]

which in combination yield

\[
\begin{align*}
\lambda &= \frac{u'(c_1) - 1}{1 - \phi p'(c_1)} \\
\lambda (c_0) &= E \left[ \frac{u'(c_1) - \phi p'(c_1)}{1 - \phi p'(c_1)} \right] > E [u'(c_1)]
\end{align*}
\]
The last equation shows that the constrained planner will borrow less than decentralized agents.

Figure 2 illustrates the constrained social planner’s intervention graphically. The left panel illustrates equilibrium in the period 0 market for debt in which $d_0$ is determined. The right panel depicts the period 1 market for debt in a constrained equilibrium in which $d_1$ is determined by binding constraints, depicted by solid vertical lines.

In each panel, the horizontal axis captures the amount of debt, and the vertical axis depicts the corresponding marginal rate of substitution between the current and next period, i.e. the price at which an agent would be willing to shift a marginal unit of consumption between the two periods. We can interpret the downward-sloping line representing the marginal rate of substitution of the emerging market agent as the demand $D$ for debt, and the flat horizontal line representing the (constant) marginal rate of substitution of international lenders as the supply $S$ of debt. The area in between the two lines represents the surplus of emerging market consumers from borrowing. We denote variables in the constrained decentralized equilibrium and in the social planner’s allocation by the superscripts $\text{con}$ and $\text{sp}$ respectively.

Since we assumed that the financial constraint in period 1 is binding, observe that each choice of debt $d_0$ in period 0 determines a specific level of consumption, the asset price and the borrowing limit in period 1. However, decentralized agents take the asset price and therefore the period 1 borrowing limit as given when they determine their period 0 borrowing – they simply choose $d_0^{\text{con}}$ such that their marginal rate of substitution equals that of their lenders, which we assumed to be 1. They end up constrained at $d_1^{\text{con}}$ in period 1.

A planner recognizes that marginally reducing period 0 borrowing to $d_0^{\text{sp}}$ creates a second-order welfare loss, illustrated by the shaded Harberger triangle in the left panel of the figure. In the following period, lower debt $d_0$ enables higher con-
sumption, pushes up the asset price and relaxes the borrowing limit in period 1 to $d_1^{eq}$. This has a first-order benefit on consumer welfare, as illustrated by the shaded trapezoid in the right panel of the figure.

The planner’s intervention therefore falls into the classical category of a second-best intervention, as described by Lipsey and Lancaster (1956). As these authors observed, the theory of the second best generally requires that a planner intervenes in all markets in which interventions are available. In the following, we broaden our focus to study more general second-best problems in which the planner has multiple policy instruments.

4.2 Taxing Borrowing Versus Supporting Asset Prices

We introduce a planner who has two policy instruments. First, she can impose a tax $\tau_d$ on borrowing $d_0$ in period 0, which is rebated as a lump sum $T_0 = \tau d_0$. This is as a macro-prudential policy instrument since it has to be used “ex-ante,” i.e. before the planner observes the shock in period 1 and before it is known whether the collateral constraint in period 1 is binding. Second, the planner can intervene to support asset prices in period 1 by imposing a subsidy $s_\theta$ on holding assets, which is financed by a lump-sum tax $T_1$. This is an “ex-post” measure that can be taken once the endowment shock is realized and it is known whether the collateral constraint on the economy is binding.

To capture the notion that government intervention in asset prices creates costly distortions, we assume that consumers have the capacity to produce counterfeit assets with a convex cost function $L(\alpha)$ that satisfies $L(0) = L'(0) = 0$ and $L'(\alpha), L''(\alpha) > 0$ for $\alpha > 0$. (Below we also analyze the case that asset price intervention is costless $L(\alpha) \equiv 0$.) Counterfeit assets do not yield any real payoffs, but allow consumers to collect government subsidies since the government cannot distinguish them from proper assets. The production costs $L(\alpha)$ are therefore a deadweight loss for society. We assume that the private sector, including international lenders, can distinguish the two and will not lend against counterfeit assets. This setup modifies the period 1 budget constraint to

$$c_1 = \tilde{e} + d_1 + (\theta_1 - \theta_2) p_1 + (\theta_2 + \alpha) s_\theta - L(\alpha) + T_1. \quad (6)$$

Remark We allow for lump-sum taxes and rebates so as to keep our focus on the efficiency implications of the planner’s policy measures, i.e. to study how they can mitigate the pecuniary externalities arising in a setting of financial amplification. If we limited our attention to revenue-neutral policies, the planner would continue to use a package of ex-ante taxes and ex-post subsidies.

The optimization problem of a representative consumer can be described as maximizing expected utility (1) subject to the budget constraints and the collateral
constraint (3), where counterfeit asset production $\alpha$ is an additional choice variable. The consumer’s optimality conditions are

\[
\begin{align*}
FOC (d_0) & : (1 - \tau_d) u'(c_0) = E [u'(c_1)] \\
FOC (d_1) & : u'(c_1) = 1 + \lambda \\
FOC (\theta_2) & : p_1 = \frac{y}{u'(c_1)} + s_\theta \\
FOC (\alpha) & : L'(\alpha) = s_\theta
\end{align*}
\]  \hspace{1cm} (7)

We combine the last two equations to obtain an asset price function

\[
p(c_1, \alpha) = \frac{y}{u'(c_1)} + L'(\alpha)
\]

which satisfies $p_c = \gamma yc_1^{\gamma-1} > 0$ and $p_\alpha = L''(\alpha) > 0$.

The collateral constraint in period 1 is binding if

\[
\hat{c} - d_0 - L(\alpha) < c^* - \phi [y + s_\theta]
\]

Observe that an asset price subsidy $s_\theta$ has two effects on the tightness of the constraint: By pushing up the asset price, it increases borrowing capacity by $\phi s_\theta = \phi L'(\alpha)$. By encouraging counterfeit production, it reduces the remaining financial net worth available to individuals by $L(\alpha)$. For small $s_\theta$ and $\alpha$, the first effect always dominates and the measure is successful in relaxing the constraint.\textsuperscript{3}

The period 1 equilibrium in the economy is determined as the solution to the implicit equation

\[
c_1 = \min \{c^*, m - L(\alpha) + \phi p(c_1, \alpha)\}
\]

Under assumption 1, this equation has a unique solution. The equilibrium in period 0 is determined from the first order condition (7).

The planner’s problem is to choose his policy instruments $(\tau_d, s_\theta)$ so as to maximize consumer welfare (1) subject to the consumer’s budget constraints, the borrowing constraint (3), and the consumer’s optimality conditions from above.

**Proposition 2 (Macroprudential Regulation and Asset Price Support)** The planner finds it optimal to impose a positive macroprudential tax $\tau_d > 0$ in period 0 if and only if there is a positive probability of binding constraints in period 1. Furthermore, the planner intervenes to prop up asset prices with $s_\theta > 0$ in any state of period 1 in which the collateral constraint is binding.

\textsuperscript{3}To see this formally, note that $\phi L'(\alpha) - L(\alpha) = \int_0^\alpha [\phi L''(\alpha) - L'(\alpha)]$ and that $L''(\alpha) > 0 = L'(\alpha)$.
Proof. After substituting the budget constraints and the price function, the planner’s problem can be formulated as

$$\max_{d_0, d_1, \alpha} E \{ u(d_0) + u(\bar{e} + d_1 - L(\alpha)) + y - d_0 - d_1 - \lambda [d_0 + d_1 - \phi p(\bar{e} + d_1 - L(\alpha), \alpha)] \}$$

The resulting optimality conditions are

$$FOC(d_0) : u'(c_0) = E[1 + \lambda]$$
$$FOC(d_1) : u'(c_1) = 1 + \lambda (1 - \phi c_p)$$
$$FOC(\alpha) : (1 + \lambda) L'(\alpha) = \lambda \phi p_\alpha$$

We combine the first two equations to obtain

$$u'(c_0) = E[u'(c_1) + \lambda \phi c_p]$$

Comparing this optimality condition to the Euler equation (7) of private agents, the planner can implement her optimal allocation by taxing period 0 borrowing at a rate

$$\tau_d = \frac{E[\lambda \phi c_p]}{u'(c_0)}.$$  \hfill (10)

This expression is positive if and only if there are some states of nature in period 1 in which \( \lambda > 0 \).

Next, the optimality condition (9) in conjunction with (8) implies that the planner sets the asset price subsidy to

$$s_\theta = \frac{\lambda \phi p_\alpha}{1 + \lambda}.$$  \hfill (11)

The subsidy is positive if and only if the collateral constraint is binding \( \lambda > 0 \). We can rewrite the expression as

$$\frac{L'(\alpha)}{L''(\alpha)} = \frac{\lambda \phi}{1 + \lambda}$$

The left-hand side of this equation is strictly increasing in \( \alpha \). The equation therefore pins down a unique optimal level of counterfeit production \( \alpha > 0 \) that is increasing in the tightness of the borrowing constraint \( \lambda \) and corresponds to an increasing asset price subsidy \( s_\theta = L'(\alpha). \) □

The tax rate in (10) reflects that a marginal increase in period 0 consumption raises debt and creates an uninternalized social cost of \( E[\lambda \phi c_p] \) by tightening the collateral constraint – the tax offsets this pecuniary externality. Similarly, the subsidy rate in (11) captures that a marginal unit spent on counterfeit assets has the positive effect of reducing the cost of binding collateral constraints by \( \lambda \phi p_\alpha \) – the planner therefore subsidizes asset holdings at that rate.
Remark When the collateral constraint is binding, both policy measures relax the constraint, which creates a first-order welfare gain, while imposing a second-order cost on the economy (i.e. a cost that is negligible for small amounts but increasing in a convex fashion): macroprudential taxation distorts optimal consumption smoothing between periods 0 and 1, and asset price subsidies come with a deadweight loss \( L(\alpha) \). This makes it optimal to use a combination of both measures.

The planner finds it optimal to mitigate binding constraints in the competitive equilibrium, but not to fully relax them as long as there are costs associated with her policy intervention. Otherwise \( \lambda = 0 \), which would imply \( \tau_d = s_g = 0 \) and which would contradict the assumption that the competitive equilibrium exhibited binding constraints.

The following corollary shows that fully relaxing binding constraints is, however, optimal if asset price intervention is costless. (This mimics the situation described in proposition ??.) To ensure a well-defined equilibrium, assume that \( L(\alpha) = 0 \) for \( \alpha \leq \bar{\alpha} \) where \( \bar{\alpha} \) is a finite constant, and \( L(\alpha) = \infty \) for \( \alpha > \bar{\alpha} \).

**Corollary 3** If \( L(\alpha) = 0 \ \forall \alpha \leq \bar{\alpha} \), the planner props up asset prices to the point where the collateral constraint is loose in all states of nature. Given the availability of a costless instrument to relax binding constraints, the first-best allocation is replicated and the optimal macroprudential tax satisfies \( \tau_d = 0 \).

**Proof.** By equation (9), \( L'(\alpha) = 0 \) implies that \( \lambda = 0 \) in all states of nature. The planner achieves this goal by setting the subsidy \( s_\theta \) such that the asset price is sufficiently high to make the constraint loose. Equation (10) then implies that \( \tau_d = 0 \). Consumers create \( \bar{\alpha} \) counterfeit assets, but this is costless since \( L(\bar{\alpha}) = 0 \).

■

In the following, we generalize our findings by discussing alternative ex-post stimulus measures to mitigate binding constraints.

### 4.3 Subsidies to Labor Supply

We modify the benchmark model of section 2 by substituting the period 1 endowment \( \tilde{e} \) by stochastic labor income \( \tilde{A}l \) and assuming that labor imposes disutility \( h(l) = \frac{\rho + \rho}{1 + \rho} \) on the consumer, where \( \rho \) represents the inverse Frisch elasticity of labor,

\[
U = u(c_0) + u(c_1) - h(l) + c_2.
\]  

We continue to assume that the planner has a macro-prudential tax instrument \( \tau_d \) on borrowing in period 0. Instead of the asset price subsidy, however, we now assume that the planner can impose a subsidy \( s_l \) on labor that is financed via lump
sum taxation $T_1 = \tilde{A}s_t$.\footnote{If we added a subsidy to labor supply to our earlier model on taxing borrowing and supporting asset prices in section 4.2, the planner would employ a positive combination of all three measures. We drop asset price subsidies in the given subsection to keep our analysis clean and transparent.} This modifies the period 1 budget constraint to

$$c_1 = (1 + s_t) \tilde{A}l + d_1 + (\theta_1 - \theta_2) p_1 - T_1.$$  

When the planner imposes a positive labor subsidy in a constrained state of nature, he induces consumers to work more, which raises their period 1 consumption and pushes up the asset price, thereby relaxing the constraint. Assumption 1 is sufficient to guarantee that the problem has a unique solution.

**Proposition 4 (Macroprudential Regulation and Labor Stimulus)** The planner imposes a positive macroprudential tax $\tau_d > 0$ in period 0 if and only if there is a positive probability of binding constraints in period 1. Furthermore, the planner stimulates labor supply with $s_l > 0$ in any state of period 1 in which the collateral constraint is binding.

**Proof.** As in the decentralized equilibrium described in section 2, the asset price is given by the function $p(c_1) = \frac{y}{u'(c_1)}$. The optimal tax on period 0 debt is derived as in the proof of proposition 2,

$$\tau_d = \frac{E[\lambda \phi p'(c_1)]}{u'(c_0)}.$$

As detailed in the appendix, the consumer’s optimality condition for labor supply is

$$h'(l) = \tilde{A}(1 + s_l) u'(c_1),$$

whereas the planner’s optimality condition is given by

$$h'(l) = \tilde{A}[u'(c_1) + \phi \lambda p'(c_1)].$$

The planner internalizes that the additional wealth created by higher labor supply pushes up asset prices at the aggregate level and relaxes the borrowing constraint by $\phi \lambda p'(c_1)$. She can induce consumers to internalize this pecuniary externality by imposing a labor subsidy

$$s_l = \frac{\lambda \phi p'(c_1)}{u'(c_1)},$$

which is positive whenever $\lambda > 0$, i.e. whenever the borrowing constraint is binding.

Combining the two expressions for optimal policy measures we find

$$\tau_d u'(c_0) = E[\lambda \phi p'(c_1)] = E[s_l u'(c_1)].$$

The planner sets the two policy measures such that the marginal cost of the tax on period 0 debt and the expected benefit of the subsidy on labor supply in period 1 are both equal to the marginal benefit of relaxing the collateral constraint, $E[\lambda \phi p'(c_1)]$.\footnotemark
4.4 Equilibrium Quantity of Debt

In the existing literature, Benigno et al. (2009, 2010ab) have pointed out that the equilibrium quantity of debt may be higher in an economy in which ex-post stimulative policy interventions are available than in the free market equilibrium without intervention. They term this phenomenon “underborrowing.” This section replicates their results and discusses the interpretation.

We observed above that a stimulus policy $s$ increases labor supply in our framework and therefore raises period 1 consumption, i.e. $\frac{dc_1}{ds} > 0$ when the financial constraint is binding. Given the period 0 Euler equation ($\Sigma\\Sigma$) of decentralized agents, higher period 1 consumption makes it optimal for consumers to also raise period 0 consumption, i.e. to borrow more for a given tax rate $\tau$. Denoting the planner’s optimal ex-post intervention in a given state of nature as $s^*$, and assuming that there are binding constraints so that $s^* > 0$ in at least some states of nature, we find unambiguously that

$$c_0|_{s=s^*} > c_0|_{s=0} \quad \text{and} \quad b_1|_{s=s^*} > b_1|_{s=0}$$

If the planner is expected to intervene ex post, financial crises will be less severe for a given amount of initial debt $b_1$; therefore it is optimal for the economy to borrow more and raise $b_1$. This replicates the results that occur under some conditions in Benigno et al. (2009, 2010ab).

By the same token, for given levels of ex-post intervention $s^*$, an increase in the macroprudential tax $\tau$ reduces the amount of period 0 consumption $c_0$ and borrowing $b_1$. Moving from a decentralized equilibrium with no policy intervention $\tau, s = 0$ to a Ramsey equilibrium where $\tau = \tau^*, s = s^*$ are chosen optimally, the equilibrium amount of debt may rise or fall, depending on whether the effects of the ex-ante policy or of the ex-post policy are stronger. However, to determine the sign of optimal policy measures, it is irrelevant whether the debt level under a Ramsey planner is higher or lower than in the decentralized equilibrium with no intervention: A Ramsey planner finds it desirable to intervene both ex-ante through macroprudential intervention and ex-post through stimulus measures, as we captured in propositions 1 and 2.

The broader point of the debate is the following: if our interest is to infer optimal policy measures in a model where a planner sets multiple policy instruments, then it can be misleading to simply compare equilibrium quantities between the free market equilibrium and the planner’s allocation. The planner in our setup finds it optimal to reduce borrowing ex-ante by imposing macroprudential taxes, but to engage in stimulus measures that relax the constraint ex post. The total effect on the equilibrium quantity borrowed consists of both the tax-induced reduction in borrowing and the stimulus-induced increase in borrowing and is of ambiguous sign.
5 Conclusions

This note provided a simple framework of optimal policy intervention in an economy that is subject to collateral-dependent borrowing constraints that make it prone to financial amplification effects. We first laid out a variety of ways in which a planner can restore the first-best equilibrium and discussed why the conditions necessary for this are not likely to be met in practice.

Then we analyzed the problem of a constrained planner who chooses the allocations in the economy while respecting the borrowing constraint and showed that there was scope for a Pareto improvement because the planner internalized the pecuniary externalities of her actions. The social planner’s allocation can be implemented as a “macroprudential” tax on borrowing in good times that reduces the severity of binding financial constraints in a later period. This intervention can be interpreted as a second-best policy in the spirit of Lipsey and Lancaster (1956).

Following the insights of the theory of the second-best that it is generally desirable to simultaneously intervene along as many dimensions as possible when engaging in second-best policies, we analyze complementary policy measures, including interventions to support asset prices and to engage in stimulus policies on labor supply. We show that it is generally optimal for policymakers to employ strictly positive amounts of ex-ante prudential and ex-post stimulus measures, up to the point where the expected marginal benefit of each measure in relaxing binding financial constraints equals its expected marginal cost. However, interventions that take place once an economy experiences binding financial constraints lead to problems of time consistency.
References


Jeanne, Olivier and Anton Korinek, 2011, "Booms, Busts and Growth", *work in progress*.


### A Mathematical Appendix

#### A.1 Derivations of the Problem with Labor Subsidy

We summarize the budget constraints as

\[
\begin{align*}
    c_0 &= (1 - \tau_d) d_0 + T_0, \\
    c_1 &= (1 + s_l) A l_1 + d_1 + (\theta_1 - \theta_2) p_1 - T_1, \\
    c_2 &= \theta_2 y - d_0 - d_1.
\end{align*}
\]  

(13)
The consumer’s optimization problem is to maximize utility (12) subject to the budget constraints (13) and the collateral constraint (3). The resulting optimality conditions are

\[
\begin{align*}
\text{FOC} (d_0) & : u' (c_0) (1 - \tau_d) = E u' (c_1) \\
\text{FOC} (d_1) & : u' (c_1) = 1 + \lambda \\
\text{FOC} (l_1) & : \tilde{A} (1 + s_1) u' (c_1) = h' (l_1) \\
\text{FOC} (\theta_2) & : p_1 = \frac{y}{u' (c_1)} := p (c_1)
\end{align*}
\]

The first-best allocation in the economy with endogenous labor supply is as characterized in proposition 1, together with an optimal choice of labor supply \( l_1 \) such that \( h_0 (l_1) = \tilde{A} \). We define the function \( l_1 = l (A) := (h')^{-1} (A) \), which satisfies \( l' (A) > 0 \).

We solve for the competitive equilibrium by backward induction and focus first at the equilibrium in periods 1 and 2 for given initial debt \( d_0 \). Consumers would like to borrow \( c^* - l (\tilde{A}) \). The collateral constraint in period 1 is loose if

\[
d_0 + c^* - l (\tilde{A}) \leq \phi y
\]

If this inequality is violated, the binding constraint determines borrowing \( d_1 = \phi p_1 - d_0 \). Period 1 consumption is then determined as the solution to the implicit equation

\[
c_1 = \tilde{A} l_1 + \phi p_1 = \tilde{A} l \left( \tilde{A} (1 + s) u' (c_1) \right) + \frac{\phi y}{u' (c_1)}
\]

Since \( \partial l_1 / \partial c_1 < 0 \), assumption 1 is a sufficient condition to guarantee that the slope of the right-hand side is less than unity and that we can rule out multiple equilibria.

Taking the asset pricing equation as determined by the private sector, the planner’s problem is to maximize consumer utility (12) subject to the budget constraints (13), the collateral constraint (3) and the optimality conditions above of decentralized agents. We can formulate the planner’s problem as

\[
\max_{d_0, d_1, l_1} E \left\{ u (d_0) + u \left( \tilde{A} l_1 + d_1 \right) - h (l_1) + y_2 - d_0 - d_1 - \lambda \left[ d_0 + d_1 - \phi p \left( \tilde{A} l_1 + d_1 \right) \right] \right\}
\]

The planner’s optimality conditions are

\[
\begin{align*}
\text{FOC} (d_0) & : u' (c_0) = E [1 + \lambda] \\
\text{FOC} (d_1) & : u' (c_1) = 1 + \lambda (1 - \phi p' (c_1)) \\
\text{FOC} (l_1) & : \tilde{A} \left[ u' (c_1) + \lambda \phi p' (c_1) \right] = h' (l_1)
\end{align*}
\]